

Angle and its Measurement

a rotation, not just an opening between two rays



Initial ray OA

the fixed starting ray, along the positive X-axis in standard position

→ rotates to

Rotation

anticlockwise gives a positive measure, clockwise gives a negative measure

→ rotates to

Terminal ray OB

where it lands decides the angle's quadrant

and the pair is ordered: (OA, OB) is not (OB, OA) , so angle AOB and angle BOA differ even for the same amount of rotation

Directed Angles *

- a directed angle AOB is the ordered pair (OA, OB) plus the rotation carrying OA onto OB
- O is the vertex, OA the initial arm, OB the terminal arm
- rotation anticlockwise is positive, rotation clockwise is negative
- zero angle: no rotation at all
- one rotation: a full turn, 360°
- straight angle: half of one rotation, 180°
- right angle: a quarter of one rotation, 90°

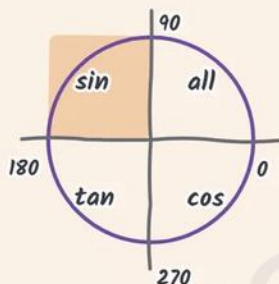
Which Quadrant, by Angle

reduce first to $0^\circ \leq \theta < 360^\circ$

$0^\circ < \theta < 90^\circ$	→ quadrant I
$90^\circ < \theta < 180^\circ$	→ quadrant II
$180^\circ < \theta < 270^\circ$	→ quadrant III
$270^\circ < \theta < 360^\circ$	→ quadrant IV
$\theta = 0^\circ, 90^\circ, 180^\circ \text{ or } 270^\circ$	→ quadrantal, terminal ray on an axis, no quadrant

Standard Position and Quadrants

- vertex at the origin, initial ray along the positive X-axis: that placement is standard position
- quadrant is decided by where the terminal ray lands, not by the angle's size
- a quadrantal angle's terminal ray sits exactly on an axis: no quadrant at all



the four quadrants, axes at 0, 90, 180, 270; each names the ratio positive there, II banded

Coterminal Angles

- angles in standard position that share the same terminal ray are coterminal
- two angles are coterminal exactly when their measures differ by an integer multiple of 360°
- that condition in radians: an integer multiple of 360° , that is $2n\pi$ radians, n an integer
- e.g. 30° , 390° and -330° are all coterminal: $390^\circ - (-330^\circ) = 720^\circ = 2 \times 360^\circ$
- to place a large or negative angle, add or subtract 360° repeatedly
- keep going until it lies between 0° and 360° , then read the quadrant

a directed angle carries a sign: the same opening between two rays reads differently depending which ray you start rotating from