

the definite integral read as area, one boundary curve at a time, then two curves subtracted

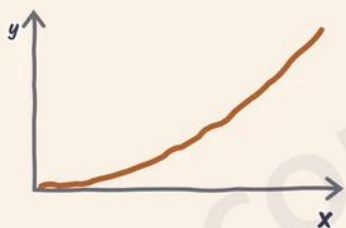
Area Under a Curve

- $y=f(x)$ continuous, $f(x) \geq 0$ on $[a,b]$: the bounded area is exactly $\int_a^b y \, dx$ from a to b
- strip picture: thin vertical strips, width dx , height y ; sum (integrate) from a to b
- same idea sideways for $x=g(y)$: thin HORIZONTAL strips, width dy , length x

Area with vertical strips (dx) and horizontal strips (dy)

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Strip	Region	Formula
dx , vertical	$y=f(x)$, X-axis, $x=a$ to $x=b$, $f(x) \geq 0$	$A = \int_a^b y \, dx$
dy , horizontal	$x=g(y)$, Y-axis, $y=c$ to $y=d$, $g(y) \geq 0$	$A = \int_c^d x \, dy$



the boundary curve only: picture it closed off below by the x -axis between $x=a$ and $x=b$, this drawing does not shade the region

choosing dx or dy is a choice of convenience: whichever variable the curve solves for naturally decides which one to integrate with

Area under a curve, dx strips

Q. Find the area bounded by the curve $y=x^2$, the X -axis, and the lines $x=0$ and $x=3$.

- $A = \int_0^3 x^2 \, dx$
- antiderivative: $x^3/3$
- apply the limits: $3^3/3 - 0^3/3 = 27/3$



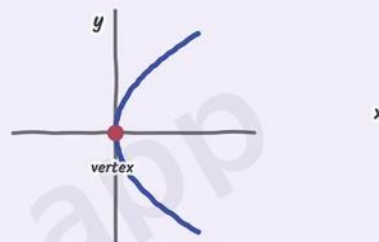
$y=x^2$ over the whole strip from $x=0$ to $x=3$, shaded: exactly the region this integral adds up

Answer 9 square units

Area under a curve, dy strips

Q. Find the area bounded by the curve $x=y^2$, the Y -axis, and the lines $y=1$ and $y=2$.

- $A = \int_1^2 x \, dy = \int_1^2 y^2 \, dy$
- antiderivative: $y^3/3$
- apply the limits: $2^3/3 - 1^3/3 = 8/3 - 1/3$



$x=y^2$ opens sideways to the RIGHT, its vertex marked at the origin: the curve and the two axes are all this drawing carries, the region between $y=1$ and $y=2$ is not drawn

Answer 7/3 square units