

the derivative as a rate, a slope and a turning point, three questions from one tool

Derivative as a rate of change

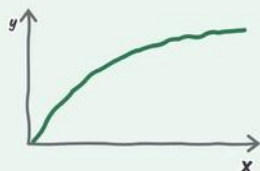
how fast y changes as x changes, at a specific point $\frac{dy}{dx} \big|_{x=x_0}$

when two quantities x and y both change with time $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

a related-rates question is this chain rule, written for two quantities depending on time

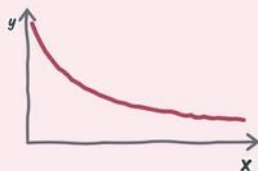
Increasing vs decreasing, tested with f' prime

Increasing on an interval



- $f'(x) \geq 0$ through the whole interval
- strictly increasing needs $f'(x) > 0$

Decreasing on an interval



- $f'(x) \leq 0$ through the whole interval
- strictly decreasing needs $f'(x) < 0$

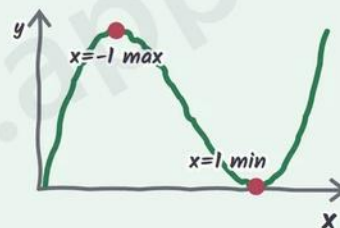
both curves rise or fall across their whole width, no turning point between: that unbroken sign is the whole test

Find where a function is increasing and decreasing

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Q. For $f(x) = x^3 - 3x$, find the intervals where f is increasing and where it's decreasing.

- 1 $f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$
- 2 $f'(x) = 0$ at $x = -1$ and $x = 1$: these split the number line into three intervals
- 3 for $x < -1$, pick $x = -2$: $f'(-2) = 3(4) - 3 = 9 > 0$, increasing there
- 4 for $-1 < x < 1$, pick $x = 0$: $f'(0) = 3(0) - 3 = -3 < 0$, decreasing there
- 5 for $x > 1$, pick $x = 2$: $f'(2) = 3(4) - 3 = 9 > 0$, increasing there



the shape of $f(x) = x^3 - 3x$: rising, a local max at $x = -1$, falling, a local min at $x = 1$, rising again

Answer

increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 1)$

Tangent and normal *

slope of the tangent at (x_0, y_0) $m = \frac{dy}{dx} \big|_{(x_0, y_0)}$

tangent line through that point $y - y_0 = m(x - x_0)$

normal line, perpendicular to the tangent, needs m not zero $y - y_0 = -\frac{1}{m}(x - x_0)$

the normal sits at right angles to the tangent, so its slope is the tangent's negative reciprocal