

Application of Integrals

a definite integral read as area, one boundary curve at a time

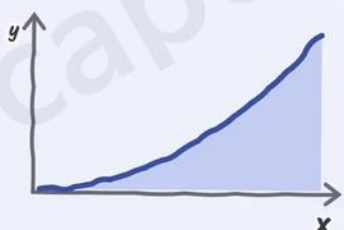
Area under a simple curve

region bounded by $y=f(x)$, the x -axis, and the lines $x=a$, $x=b$, with $f(x) \geq 0$ on $[a,b]$

$$\text{Area} = \int_a^b y \, dx$$

if the curve crosses the x -axis at $x=c$ inside $[a,b]$, split there first

$$\text{Area} = \left| \int_a^c y \, dx \right| + \left| \int_c^b y \, dx \right|$$



the shaded region between the curve and the x -axis:
this is the area the integral of $y \, dx$ returns

a definite integral itself can come out negative when the curve sits below the x -axis, but an area never can, so that portion always gets wrapped in absolute value bars

Reading a definite integral as area *

- a definite integral is the signed value the fundamental theorem returns, not automatically an area
- calling it an area needs the sign checked first; any negative part is made positive
- a shaded figure shows the exact region; an unshaded boundary curve still needs it imagined

Area bounded by a simple curve

Q. Find the area bounded by $y=x^2$, the x -axis, and the lines $x=1$ and $x=3$.

- 1 Area = the integral of $x^2 \, dx$ from $x=1$ to $x=3$
- 2 antiderivative: $x^3/3$
- 3 apply the limits: $(3^3/3) - (1^3/3) = 27/3 - 1/3$



$y=x^2$ shaded between $x=1$ and $x=3$, the exact region this integral measures

Answer **$26/3$ square units**

Area bounded by a curve and the y -axis

region bounded by $x=f(y)$, the y -axis, and the lines $y=c$, $y=d$, with $f(y) \geq 0$ on $[c,d]$

$$\text{Area} = \int_c^d x \, dy$$

this is the same idea as the x -axis version, just read sideways: sum up thin horizontal strips instead of thin vertical ones