

one slope reused six ways: a tangent line, a rate, an estimate, two named theorems, and a turning point

Tangent and Normal *

slope of the tangent at $P(x_1, y_1)$, m
 $= dy/dx$ evaluated there

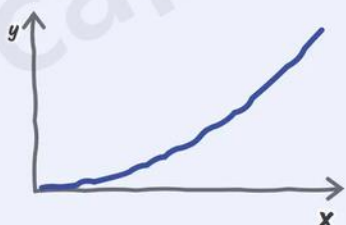
$$m = \left(\frac{dy}{dx} \right)_{(x_1, y_1)}$$

slope of the normal at the same point, perpendicular to the tangent

$$m' = -\frac{1}{m}, m \neq 0$$

equation of the tangent at $P(x_1, y_1)$ $y - y_1 = m(x - x_1)$

equation of the normal at $P(x_1, y_1)$ $y - y_1 = m'(x - x_1)$



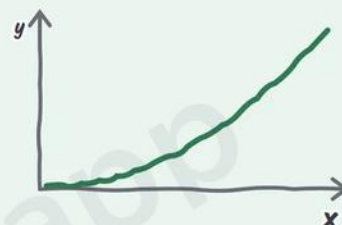
a rising curve: at every point on it the derivative gives the slope of the tangent there, the tangent line itself is not drawn

holds for an implicit or a parametric curve too, once dy/dx is found there; the tangent and normal formulas themselves never change

Tangent and normal to a cubic

Q. Find the equations of the tangent and normal to $y = 2x^3 - x^2 + 2$ at the point $(1/2, 2)$.

- ① $dy/dx = 6x^2 - 2x$
- ② at $x = 1/2$: $m = 6(1/4) - 2(1/2) = 3/2 - 1 = 1/2$
- ③ tangent: $y - 2 = (1/2)(x - 1/2)$, i.e. $2y - 4 = x - 1/2$, so $2x - 4y + 7 = 0$
- ④ normal slope $m' = -1/m = -2$: $y - 2 = -2(x - 1/2)$, i.e. $y - 2 = -2x + 1$



a rising, concave-up curve: the local shape of $y = 2x^3 - x^2 + 2$ near $x = 1/2$, where this tangent is taken

Answer

tangent: $2x - 4y + 7 = 0$; normal: $2x + y - 3 = 0$

Special tangent cases

- a horizontal tangent happens exactly where $dy/dx = 0$: the curve momentarily stops rising or falling
- a vertical tangent happens where $dx/dy = 0$ instead, since dy/dx itself would be undefined there
- for an implicit curve, differentiate implicitly for dy/dx in x and y , then substitute the point

Derivative as a Rate Measure

instantaneous rate of change of y w.r.t. x

$$\frac{dy}{dx}$$

when two quantities x and y both change with time t

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

a rate-measure question is this same chain rule, just spoken in words: 'how fast is the area growing' means find dA/dt via dA/dr times dr/dt