

a list that grows by the same step every time, and the two formulas that follow from it

A sequence

3, 7, 11, 15, ...

numbers in a fixed order, each with a position

→ then

Test the gaps

$$7-3 = 11-7 = 15-11 = 4$$

every consecutive difference is the same

→ then

It is an A.P.

$$a = 3, d = 4$$

first term and common difference describe it fully

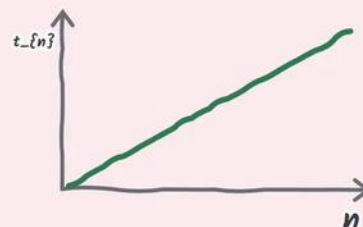
the whole chapter rests on those two numbers: once a and d are known every term and every sum can be written down without listing anything

The definition, stated carefully

- a sequence is an A.P. when the difference between any term and the one before it is constant
- that constant is the common difference d , and $d = t_2 - t_1 = t_3 - t_2$ and so on
- d is found by later term minus earlier term, never the other way round
- d may be positive, giving a rising list, negative, giving a falling one, or zero, giving a constant list
- checking one pair of terms is not enough. A list is an A.P. only if every consecutive difference agrees

Why the n -th term has n minus 1 in it

- the first term has had no step added to it yet, so $t_1 = a$
- the second term has had one step added, $t_2 = a + d$, and the third has had two
- so the term at position n has had $(n-1)$ steps, giving $t_n = a + (n-1)d$
- this is the single most useful sanity check in the chapter: substitute $n = 1$ and the formula must return a



plotting each term against its position gives points lying on a straight line, because a fixed step is added every time: the steepness of that line is d , and the height at $n = 1$ is a

The formulas this chapter is built on

the n -th term, counting from the first $t_n = a + (n-1)d$

sum of the first n terms $S_n = \frac{n}{2} [2a + (n-1)d]$

the same sum when the last term is known $S_n = \frac{n}{2} (a + t_n)$

recovering a single term from the sums $t_n = S_n - S_{n-1}$

sum of the first n natural numbers $S_n = \frac{n(n+1)}{2}$

the second and third are the same formula: substituting $t_n = a + (n-1)d$ into the third gives the second, so learn one and derive the other