

points on a line measured by numbers, betweenness, and the axioms everything later rests on

The undefined terms, and why they stay undefined

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- point, line and plane are taken as undefined. Every definition must rest on something, and these are where the resting stops
- an axiom is a statement accepted without proof, and a theorem is one proved from the axioms
- a line has no endpoints and extends for ever in both directions
- a ray has one endpoint and goes on for ever the other way, and a segment has two endpoints and a definite length
- so only a segment has a length, which is why questions measure segments and never lines
- collinear points all lie on one line, and points that do not are non collinear

Co-ordinates on a line, and distance

- each point of a line can be matched with exactly one real number, called its co-ordinate
- the distance between two points is the positive difference of their co-ordinates, written $d(A,B) = |x - y|$
- the modulus bars matter: distance is never negative, so the smaller co-ordinate is subtracted from the larger
- so $d(A,B)$ and $d(B,A)$ are the same number, unlike a difference which changes sign
- if A has co-ordinate -3 and B has 5, then $d(A,B) = |-3 - 5| = 8$
- two segments are congruent when their lengths are equal, written with the $\cong$ symbol



points marked along a line with their co-ordinates written beneath: the distance between any two of them is the gap counted along the line, which is the positive difference of the two numbers however they are ordered

Betweenness, stated exactly

- point B lies between A and C when all three are collinear and $d(A,B) + d(B,C) = d(A,C)$
- both conditions are needed: three points can satisfy the distance sum only if they are on one line, and collinearity alone does not fix the order
- this is written $A - B - C$, and it is the test a question actually wants when it asks whether a point lies between two others
- the midpoint of a segment is the point of it that is equidistant from both ends
- every segment has exactly one midpoint, and its co-ordinate is the average of the two end co-ordinates
- if the sum of the two smaller distances exceeds the largest, the three points form a triangle instead