

n independent trials, each success or failure with the same chance, and every count from 0 to *n* has its own term in an expansion

Bernoulli Trials

- trials are Bernoulli if each has two outcomes (success, failure) with the same success probability
- success/failure are labels: calling even a success fixes $p=1/2$, three of six faces are even
- trials must also be independent, so that fixed success probability genuinely applies unchanged every time
- notation: p is success probability, $q=1-p$ is failure probability, both constant across n trials

sampling **WITH** replacement keeps p constant and stays Bernoulli. Sampling **WITHOUT** replacement changes p after every draw, and the moment it does, the trials stop being Bernoulli trials

three Bernoulli trials, success probability p , failure probability $q=1-p$

From three trials to the general term

the sample space is SSS, SSF, SFS, FSS, SFF, FSF, FFS, FFF, and X counts the successes, so X takes values 0, 1, 2, 3

$$P(X=0)=P(FFF)=q^3, P(X=3)=P(SSS)=p^3$$

$$P(X=1)=P(SFF)+P(FSF)+P(FFS)=3pq^2, \text{ since three orderings give exactly one success}$$

$$P(X=2)=P(SSF)+P(SFS)+P(FSS)=3p^2q, \text{ since three orderings give exactly two successes}$$

these four probabilities, $q^3, 3q^2p, 3p^2q, p^3$, are exactly the four terms of the binomial expansion $(q+p)^3$

generalising to n trials, x successes and $(n-x)$ failures can occur in ${}^nC_x$ distinct orderings, and each ordering has probability $p^x q^{n-x}$

$P(x \text{ successes in } n \text{ trials}) = {}^nC_x p^x q^{n-x}$, the $(x+1)$ th term of the expansion of $(q+p)^n$

Deciding whether draws are Bernoulli trials

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Q. six balls are drawn successively from an urn of 7 red and 9 black balls. Are the draws Bernoulli trials (i) with replacement, (ii) without replacement?

- with replacement: the urn resets after every draw, so $P(\text{red}) = \frac{7}{16}$ on all six draws
- the probability of success stays fixed across all six draws, satisfying both Bernoulli conditions
- without replacement: $P(\text{red})$ is $\frac{7}{16}$ first, then $\frac{6}{15}$ if red was drawn, or $\frac{7}{15}$ if black

Answer

with replacement, the draws **ARE** Bernoulli trials. Without replacement, they are **NOT**, since the probability of success changes from draw to draw

the number of trials being fixed is not enough on its own, the probability of success has to stay fixed too, trial after trial

Binomial Distribution

probability function of X , for $x=0,1,\dots,n$

$$P(x) = {}^nC_x p^x q^{n-x}$$

notation: X follows a binomial distribution with parameters n and p

$$X \sim B(n, p)$$

the two conditions the trials must satisfy

n fixed, trials independent

for given n and p the **WHOLE** distribution is fixed, every $P(x)$ for $x=0$ to n comes straight out of this one formula