



expanding $(a+b)^n$ without multiplying it out by hand

What the theorem gives

- $(a+b)^n$ expands into $n+1$ terms: a coefficient, a power of a , a power of b
- a 's power falls n to 0 , b 's rises 0 to n : they add to n
- the coefficients are the combinations ${}^nC_0$ to ${}^nC_n$, the numbers counted last chapter

$$(a+b)^2$$

$$\begin{array}{c} \swarrow \quad \downarrow \quad \searrow \\ a^2 \quad 2ab \quad b^2 \end{array}$$

the three terms of $(a+b)^2$, powers of a falling, powers of b rising, coefficients 1, 2, 1

Pascal's triangle *

each entry is the sum of the two entries above it

$${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$$

the two outer entries of every row

$${}^nC_0 = {}^nC_n = 1$$



row n above, row $n+1$ built below it: the arrow runs the way the triangle grows, and each entry of row $n+1$ is the sum of the two entries of row n that flank it

The binomial theorem

full expansion

$$(a+b)^n = \sum_{k=0}^n {}^nC_k a^{n-k} b^k$$

the general term, $(k+1)$ th in the sum

$$T_{k+1} = {}^nC_k a^{n-k} b^k$$

T_{k+1} is indexed from $k=0$, so the FIRST term of the expansion is T_1 ($k=0$), not T_0

The first four rows of Pascal's triangle

$n=0$

1

$n=1$

1 1

$n=2$

1 2 1

$n=3$

1 3 3 1

each row starts and ends in 1. every number in between is the sum of the two numbers above it

