



the laws that shrink an expression, and the map that does it without algebra



Two values, three operations, one algebra

- a **Boolean variable** takes exactly two values, written 0 and 1 or false and true, and there is nothing in between
- the three operations are AND written as a product, OR written as a sum, and NOT written as a bar over the term
- every law below can be proved by **truth table**, which is the method the paper accepts when it says verify
- the algebra is not ordinary arithmetic: $1 + 1$ is 1 here, because $+$ means OR

The postulates and the basic theorems

operations with 0 and 1

$$A + 0 = A, A + 1 = 1, A \cdot 0 = 0, A \cdot 1 = A$$

complements

$$A + A' = 1, A \cdot A' = 0$$

idempotence

$$A + A = A, A \cdot A = A$$

involution

$$(A')' = A$$

commutativity

$$A + B = B + A, A \cdot B = B \cdot A$$

associativity

$$A + (B + C) = (A + B) + C$$

distributivity

$$A \cdot (B + C) = A \cdot B + A \cdot C, A + B \cdot C = (A + B) \cdot (A + C)$$

absorption

$$A + A \cdot B = A, A \cdot (A + B) = A$$

De Morgan

$$(A \cdot B)' = A' + B', (A + B)' = A' \cdot B'$$

the second distributive law has no arithmetic counterpart, which is why it is the one most often disbelieved

Duality, which halves what has to be learnt



- swap every AND with OR and every 0 with 1, leave the variables alone, and a true statement stays true
- so $A + 0 = A$ has the dual $A \cdot 1 = A$, and the two absorption laws are duals of each other
- the **complement** of an expression is not its dual: complementing also bars every variable, which is De Morgan
- in the exam, proving one of a dual pair is normally enough to claim the other, but say so explicitly

