

every point the same distance from one centre, in three equivalent equations and one tangent formula

## A Circle is a Locus

- a circle is the set of all points in a plane equidistant from one fixed point
- that fixed point is the centre, that fixed distance is the radius
- the standard form  $x^2+y^2=r^2$  is the circle centred at the origin with radius  $r$
- the centre-radius form  $(x-h)^2+(y-k)^2=r^2$  is the same circle recentred at  $(h,k)$
- it's built from  $CP=r$  and the distance formula
- the diameter form  $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$  comes from the angle-in-a-semicircle theorem
- if  $A,B$  are the ends of a diameter and  $P$  is any point on the circle
- then  $\angle APB=90^\circ$ , so  $AP \perp BP$
- the product of their slopes is  $-1$

## The Three Forms \*

standard

$$x^2+y^2=r^2$$

centre-radius

$$(x-h)^2+(y-k)^2=r^2$$

diameter, ends  $A(x_1,y_1)$ ,  $B(x_2,y_2)$

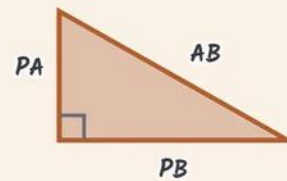
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$



distance measured between two points: exactly the idea  $CP=r$  captures, a fixed gap between the centre and every point on the circle

Q. Find the equation of the circle with  $A(2,-3)$  and  $B(-3,5)$  as ends of a diameter

- diameter form:  $(x-2)(x+3)+(y+3)(y-5)=0$
- expand:  $x^2+x-6+y^2-2y-15=0$

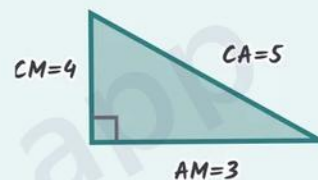


triangle  $APB$  with the right angle at  $P$ :  $A,B$  are the diameter's ends, Thales' theorem

Answer  $x^2+y^2+x-2y-21=0$

Q. Centre  $(3,-4)$ , and the line  $3x-4y-5=0$  cuts the circle in a chord of length 6. Find the equation.

- the perpendicular distance from the centre to the chord:  $CM = \frac{|3(3)-4(-4)-5|}{\sqrt{3^2+4^2}} = \frac{|9+16-5|}{5} = \frac{20}{5} = 4$
- half the chord  $AM=3$ , so by Pythagoras on right triangle  $AMC$ :  $CA^2=CM^2+AM^2=16+9=25$ , giving radius  $CA=5$
- centre-radius form:  $(x-3)^2+(y+4)^2=25$



right triangle  $CMA$ : perpendicular from centre to chord ( $CM=4$ ), half-chord ( $AM=3$ ), radius ( $CA=5$ )

Answer  $x^2+y^2-6x+8y=0$