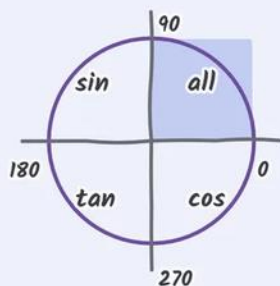


points become number pairs, so distance, division and slope all turn into arithmetic

The plane, and how a point is named

- two perpendicular number lines meet at the origin $O(0, 0)$, the horizontal one called the X axis and the vertical one the Y axis
- a point is written as an ordered pair (x, y) , the x co-ordinate always first, and $(3, 5)$ is a different point from $(5, 3)$
- x is the distance from the Y axis and y the distance from the X axis, each carried with its sign
- the two axes cut the plane into four quadrants, numbered anticlockwise starting from the upper part on the side where both are positive
- a point on the X axis has $y = 0$, and a point on the Y axis has $x = 0$, which is how such conditions are turned into equations



the four quadrants formed by the two axes, with the first one highlighted: signs run $(+, +)$ there, then $(-, +)$, then $(-, -)$, then $(+, -)$ as you travel anticlockwise

The four formulas the chapter runs on

distance between two points $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

midpoint of the segment joining them $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

point dividing the segment in the ratio $m:n$ $P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$

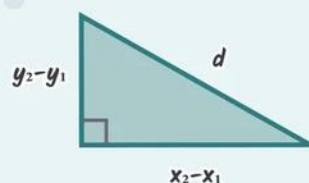
centroid of a triangle from its vertices $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$

slope of the line through two points $m = \frac{y_2 - y_1}{x_2 - x_1}$

the midpoint formula is the section formula with $m:n$ equal to $1:1$, so there is really one formula here rather than two

Why the distance formula is the Pythagoras theorem

- join the two points and complete a right angled triangle with one side horizontal and one vertical
- the horizontal side has length $|x_2 - x_1|$ and the vertical side $|y_2 - y_1|$
- the segment joining the points is the hypotenuse of that triangle
- applying $a^2 + b^2 = c^2$ and taking the square root gives the formula exactly
- because both differences are squared, the order of the two points does not matter and the distance is never negative



the horizontal gap and the vertical gap between two points forming the two shorter sides, with the join between the points as the hypotenuse: the distance formula is this triangle written out