

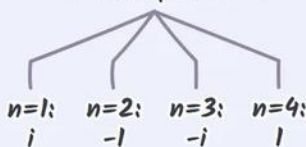
Complex Numbers and Quadratic Equations

a second axis for numbers, so every quadratic finally has a full set of roots

Why i exists *

- $x^2+1=0$ has no solution among real numbers, since a real square is never negative
- define i so that $i^2=-1$: a new kind of number, built to fill exactly this gap
- $z=a+bi$, with a and b real: a the real part, b the imaginary part
- when $b=0$, z is purely real. when $a=0$ and $b \neq 0$, z is called purely imaginary
- two complex numbers are equal only when both parts match: $a=c$ and $b=d$

i to the power n



the four values i , -1 , $-i$, 1 that i^n cycles through, one for each remainder n leaves on division by 4

Algebra of complex numbers

operation	rule
addition	$(a+bi)+(c+di)=(a+c)+(b+d)i$
subtraction	$(a+bi)-(c+di)=(a-c)+(b-d)i$
multiplication	$(a+bi)(c+di)=(ac-bd)+(ad+bc)i$
conjugate	$\bar{z}=a-bi$
modulus	$ z =\sqrt{a^2+b^2}$
division	$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{c^2+d^2}$

multiplication expands like binomials, then replaces i^2 with -1 . Division clears i by multiplying by the denominator's own conjugate

Conjugate and modulus

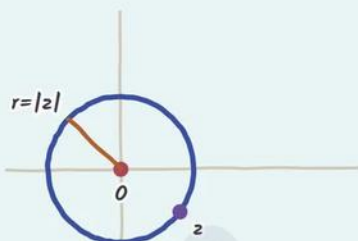
conjugate of $z=a+bi$: flip the sign of the imaginary part

$$\bar{z}=a-bi$$

a number times its own conjugate is always real

$$z\bar{z}=a^2+b^2$$

modulus: distance of z from the origin $|z|=\sqrt{a^2+b^2}$



every point at distance r from the origin: fixing $|z|=r$ puts z somewhere on this circle

z times its conjugate equals the modulus squared, so the conjugate is also the fast route to a modulus

Powers of i cycle every 4

n leaves remainder 1

$$i^1 = i$$

n leaves remainder 2

$$i^2 = -1$$

n leaves remainder 3

$$i^3 = -i$$

n leaves remainder 0

$$i^4 = 1$$

to find i to any power n , divide n by 4 and match the remainder against these four cases