



no real number squares to -1 , so the number system was widened until one did

The imaginary unit *

- $x^2+1=0$ has no solution among the real numbers: no real number squares to -1
- define i (iota) by $i=\sqrt{-1}$, so $i^2=-1$. This is the imaginary unit
- a number of the form bi , $b \in \mathbb{R}$, $b \neq 0$, is called an imaginary number, e.g. $5i$, $\frac{2}{7}i$
- a complex number is $z=a+ib$, $a, b \in \mathbb{R}$
- a is the real part $\text{Re}(z)$, b is the imaginary part $\text{Im}(z)$
- the set of all such z is $\mathbb{C}$, and $\mathbb{R} \subset \mathbb{C}$

Equality and conjugate

equality

$$a+ib=c+id \Leftrightarrow a=c \text{ and } b=d$$

conjugate of $z=a+ib$

$$\bar{z}=a-ib$$

$z=a+ib$ behaves as the ordered pair (a, b) , real part first, imaginary second, never merged into one term. complex numbers cannot be ordered: there is no meaning to $i < 4$. Equality is the only comparison that exists

Solving by equality of complex numbers

Q. if $7a+i(3a-b)=21-3i$, find a and b

- match real parts: $7a=21$, so $a=3$
- match imaginary parts: $3a-b=-3$, so $3(3)-b=-3$

$$\begin{bmatrix} 7a & 21 \\ 3a-b & -3 \end{bmatrix}$$

real parts on the top row, imaginary below: each row is one equation

Answer $a=3$, $b=12$

Properties of the conjugate

Double conjugate

$$z=\bar{\bar{z}}$$

z real

$$z=\bar{z} \Leftrightarrow z \text{ is purely real}$$

z imaginary

$$z=-\bar{z} \Leftrightarrow z \text{ is purely imaginary}$$

Modulus link

$$z \cdot \bar{z} = a^2 + b^2$$

The four operations *

addition

$$(a+ib)+(c+id)=(a+c)+i(b+d)$$

subtraction

$$(a+ib)-(c+id)=(a-c)+i(b-d)$$

multiplication

$$(a+ib)(c+id)=(ac-bd)+i(ad+bc)$$

division, multiply top and bottom by $\bar{z}_2$

$$\frac{z_1}{z_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2}$$

multiplication is not "multiply the real parts, multiply the imaginary parts": every one of the four cross terms ac , adi , bci , bdi^2 must be kept, and $i^2=-1$ folds the last one back into the real part

Multiplying two complex numbers

Q. find $z_1 z_2$ if $z_1=2+3i$ and $z_2=3-2i$

- $z_1 z_2 = (2+3i)(3-2i) = 2(3-2i) + 3i(3-2i)$
- $= 6 - 4i + 9i - 6i^2$
- $i^2 = -1$, so $-6i^2 = +6$: $= 6 - 4i + 9i + 6$

Answer $12+5i$

