

one ratio, focus distance over directrix distance, and its value alone decides parabola, ellipse or hyperbola

The Focus-Directrix Definition *

- a conic section is the locus of a point P at a fixed distance ratio
- S is a fixed point, the focus; d is a fixed line, the directrix
- that ratio, SP to P's distance from d, stays CONSTANT for every point P
- that constant ratio SP/PM is the eccentricity e; the defining relation is $SP = e \cdot PM$
- $e=1$ gives a parabola; $0 < e < 1$ gives an ellipse; $e > 1$ gives a hyperbola
- these curves are literally slices of a double cone
- a plane perpendicular to the axis gives a circle
- one parallel to a generator gives a parabola
- one oblique but not parallel to any generator gives an ellipse
- one parallel to the axis gives a hyperbola

Vocabulary

axis = the line about which the conic is symmetric

vertex = where the conic meets its own axis

focal chord = a chord through the focus

latus rectum = the focal chord perpendicular to the axis

focal distance = the distance from a point on the conic to the focus

double ordinate = a chord through a point on the curve, perpendicular to the axis

Conic Type, by Eccentricity

one number, e, tells you which curve you are looking at

$e=0$	→ circle (the special ellipse case)
$0 < e < 1$	→ ellipse
$e=1$	→ parabola
$e > 1$	→ hyperbola

Standard Parabola: $y^2 = 4ax$

- focus S(a,0), directrix $x+a=0$, vertex at the origin, with $a > 0$
- setting $SP = PM$ and squaring collapses to $y^2 = 4ax$
- the algebra loses every square root because both sides are distances squared
- the curve is symmetric about the X-axis: every $x > 0$ gives two y-values, negatives of each other
- it exists only for $x \geq 0$, since $y^2 \geq 0$ forces $4ax \geq 0$

The Four Standard Parabolas *

Feature	$y^2 = 4ax$	$x^2 = 4by$
Focus	(a,0)	(0,b)
Directrix	$x+a=0$	$y+b=0$
Axis	$y=0$	$x=0$
Latus rectum ends	$(a, \pm 2a)$	$(\pm 2b, b)$
Length of latus rectum	4a	4b
Tangent at vertex	$x=0$	$y=0$
Focal distance of P(x_1, y_1)	x_1+a	y_1+b

$y^2 = -4ax$ opens left and $x^2 = -4by$ opens down: the sign flips only the direction the curve opens, everything else in the row carries over unchanged