

circle, parabola, ellipse, hyperbola, one number apart

Conic sections, and where they come from

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- each curve is points P obeying one distance rule: a fixed focus, a fixed directrix
- eccentricity $e = \frac{PF}{PM}$, focus distance over directrix distance, is the SINGLE deciding number
- $e=0$: circle. $0 < e < 1$: ellipse. $e=1$: parabola. $e > 1$: hyperbola, and these four ranges never overlap



distance measured between two points on a line: PF works exactly this way, and PM, the distance to the directrix, is a perpendicular distance measured the same manner

Eccentricity: the limiting cases

$$e = \frac{PF}{PM}$$

$e=0$	→ circle, foci merge at the centre	$e=0$
$0 < e < 1$	→ ellipse, bounded closed curve	$0 < e < 1$
$e=1$	→ parabola, one focus, one directrix, one branch	$e=1$
$e > 1$	→ hyperbola, two unbounded branches	$e > 1$

a circle is the special ellipse with $a=b$, which is why its two foci merge. These four ranges never overlap, and the fork two pages on sorts a given e by them

Circle: centre-radius and general form

centre (h,k) , radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

general form, centre and radius drop out of g,f,c

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

centre

$$(-g, -f)$$

radius, real only when $g^2 + f^2 - c > 0$

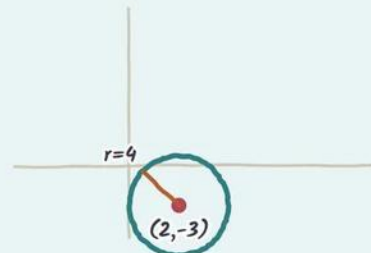
$$r = \sqrt{g^2 + f^2 - c}$$

if $g^2 + f^2 - c = 0$ the circle shrinks to a single point, and if it's negative there's no real circle at all

Worked: centre and radius from the general equation

Q. find the centre and radius of the circle $x^2 + y^2 - 4x + 6y - 3 = 0$

- match against $x^2 + y^2 + 2gx + 2fy + c = 0$: $2g = -4$ so $g = -2$, $2f = 6$ so $f = 3$, $c = -3$
- centre = $(-g, -f) = (2, -3)$
- radius: $r = \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + 3^2 - (-3)}$
- $= \sqrt{4 + 9 + 3} = \sqrt{16} = 4$



the circle centred at $(2, -3)$ with radius 4, the two numbers found beside it

Answer centre $(2, -3)$, radius 4