

drawn without lifting the pen: three conditions, and dropping any one breaks it

Continuity at a point $x=a$ *

condition 1: f is defined at a , and on an open interval around it

$$f(a)$$

condition 2: the limit as x approaches a exists

$$\lim_{x \rightarrow a} f(x)$$

condition 3: that limit equals the function's own value at a

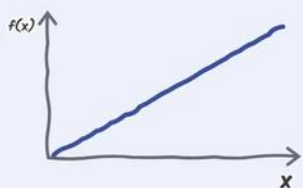
$$\lim_{x \rightarrow a} f(x) = f(a)$$

drop any one of the three and continuity fails at that point, even if the other two hold perfectly. An equivalent restatement: f is continuous at a if $\lim_{h \rightarrow 0} [f(a+h) - f(a)] = 0$

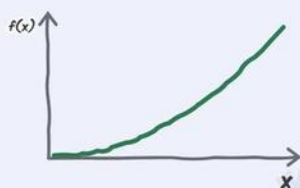
Continuity from the left and from the right

- f is continuous from the right at a if $\lim_{x \rightarrow a^+} f(x) = f(a)$
- f is continuous from the left at a if $\lim_{x \rightarrow a^-} f(x) = f(a)$
- continuous from both sides at a is exactly continuous at a
- continuity over an interval: continuous at every interior point
- plus right-continuous at a closed left endpoint
- and left-continuous at a closed right endpoint

What an unbroken curve looks like



drawn without lifting the pen: this is what continuous looks like, at every point along it



a polynomial curve is continuous at every real x , whatever its degree

Worked: continuity of a three-piece function

Q. $f(x) = 3x+1$ for $x < 2$, $= 7$ for $2 \leq x < 4$, $= x^2 - 8$ for $x \geq 4$.

Test continuity at $x=2$ and $x=4$

- at $x=2$: $f(2) = 7$. LHL = $\lim_{x \rightarrow 2^-} (3x+1) = 3(2)+1 = 7$.
RHL = $\lim_{x \rightarrow 2^+} (7) = 7$
- LHL = RHL = $f(2) = 7$, so f is continuous at $x=2$
- at $x=4$: $f(4) = 4^2 - 8 = 8$. LHL = $\lim_{x \rightarrow 4^-} (7) = 7$.
RHL = $\lim_{x \rightarrow 4^+} (x^2 - 8) = 4^2 - 8 = 8$
- LHL = 7 but RHL = 8, so $\lim_{x \rightarrow 4} f(x)$ does not exist

Answer

continuous at $x=2$, discontinuous at $x=4$ (a jump discontinuity)

Functions that are always continuous

constant and power functions

$f(x) = k$ and $f(x) = x^n$ are continuous on all of $\mathbb{R}$

every polynomial

built from sums and products of continuous pieces, so it stays continuous

$\sin x$ and $\cos x$

continuous at every real x , no exceptions

exponential and logarithm

a^x continuous on $\mathbb{R}$, $\log_b x$ continuous on $(0, \infty)$