

an antiderivative evaluated between two limits, and eight properties that dodge the antiderivative entirely

Definite Integral as a Limit of a Sum

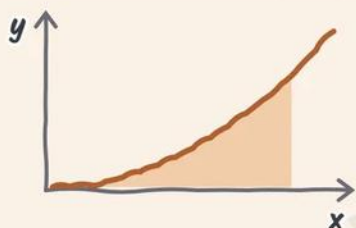
- split $[a,b]$ into n equal strips, width $h=(b-a)/n$
- sum f at each strip edge, times strip width
- as $n \rightarrow \infty$, that sum becomes $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h \sum_{r=0}^{n-1} f(a+rh)$
- this limit-of-a-sum IS the area; the theorem below skips the limit

Fundamental Theorem of Integral Calculus

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if g is any antiderivative of f , $g'(x)=f(x)$

$$\int_a^b f(x) dx = g(b) - g(a)$$



the shaded strip between the two limits is exactly what the integral totals

find the same indefinite integral as always, drop +c entirely, and plug in upper limit minus lower limit; the constant would cancel out anyway

Evaluate by the fundamental theorem

Q. Evaluate $\int_1^3 x^2 dx$.

① antiderivative: $g(x) = x^3/3$

② $g(3) - g(1) = 27/3 - 1/3$

Answer 26/3

Properties I to IV

Property I: equal limits give zero area $\int_a^a f(x) dx = 0$

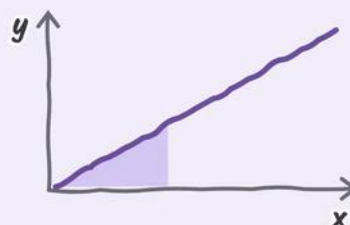
Property II: swapping the limits flips the sign

$$\int_a^b f(x) dx = -\int_b^a f(x) dx$$

Property III: the integration variable's name never matters

$$\int_a^b f(x) dx = \int_a^b f(t) dt$$

Property IV: split at any c between a and b $\int_a^b f = \int_a^c f + \int_c^b f$



splitting the interval: the shaded left piece plus the unshaded right piece make the whole

property IV runs in reverse too: split at a break or a kink BEFORE integrating