

Determinants and Matrices

a single number squeezed out of a square array, and the array itself as an object you can add, scale and multiply

A Determinant, Order 2 and Order 3

- a determinant of order 2 is the square array a, b, c, d between vertical bars
- its value is $ad - bc$
- a determinant of order 3 has 9 elements in 3 rows and 3 columns
- denoted a_{ij} for the entry in row i , column j
- a determinant is written with a Capital letter or the symbol Δ
- its array sits between vertical BARS, not square brackets
- brackets are the notation for a matrix instead
- there are six ways to expand a 3×3 determinant, one per row and per column
- expanding along any of them gives the same value

Expansion Along the First Row *

order 2

$$|a \ b; \ c \ d| = ad - bc$$

order 3, by row 1

$$D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

a 3×3 array: every entry named by the row and column it sits in, a_{ij} in row i and column j

the expansion by row or column always alternates sign, plus minus plus, starting from a_{11}

Minors and Cofactors

- the minor M_{ij} of element a_{ij} is the determinant left after deleting row i , column j
- the cofactor $C_{ij} = (-1)^{i+j}M_{ij}$: same value as the minor, sign flipped when $i+j$ is odd
- signs alternate in a checkerboard starting with plus at position $(1,1)$
- for a 2×2 determinant a, b, c, d : the minor of a is d , of b is c
- the minor of c is b , the minor of d is a

Q. Find the minors and cofactors of every element of $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

- minor of $a_{11}=1$ is the surviving entry $a_{22}=4$, so $M_{11}=4$ and $C_{11}=(-1)^{1+1}(4)=4$
- minor of $a_{12}=2$ is $a_{21}=3$, so $M_{12}=3$ and $C_{12}=(-1)^{1+2}(3)=-3$
- minor of $a_{21}=3$ is $a_{12}=2$, so $M_{21}=2$ and $C_{21}=(-1)^{2+1}(2)=-2$
- minor of $a_{22}=4$ is $a_{11}=1$, so $M_{22}=1$ and $C_{22}=(-1)^{2+2}(1)=1$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

the 2×2 array whose minors and cofactors are computed here; a determinant of it takes bars, not these brackets

Answer $C_{11}=4, C_{12}=-3, C_{21}=-2, C_{22}=1$