

one number per square array, and it decides whether the array can be undone

A determinant, defined *

order 2, entries a, b, c, d read across each row

$$|A| = ad - bc$$

order 3, general entry in row i column j

$$a_{ij}, 1 \leq i, j \leq 3$$

only a square array has one at all

$$|A|_{n \times n}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

a 2x2 array: two entries multiply along the main diagonal, two along the other

a 3x3 array, every entry named by its row and column

a determinant is one number computed from a square array, not the array itself: writing $|A|$ instead of A already says a single value is coming out

Properties that shortcut the arithmetic

Swap two rows

the determinant flips sign

Two rows identical

the determinant is exactly 0

Common factor in a row

pull it out in front of the determinant

Triangular array

the determinant is just the product of the diagonal entries

adding a multiple of one row to another (row2 becomes row2 plus k times row1) leaves the determinant completely unchanged

More properties worth knowing

- swapping two columns behaves exactly like swapping two rows: the sign flips
- multiply one row by k: the whole determinant scales by k, once, not per entry
- adding a multiple of one column to another leaves the determinant unchanged too, like rows
- the determinant of A transpose equals the determinant of A itself: $|A^T| = |A|$

Expand a 3x3 determinant

Q. Evaluate the determinant of the array with rows (2, 3, 1), (1, -1, 2) and (0, 4, -3).

- expand along row 1: $|A| = 2M_{11} - 3M_{12} + 1M_{13}$, sign pattern (+, -, +)
- $M_{11} = (-1)(-3) - (2)(4) = 3 - 8 = -5$
- $M_{12} = (1)(-3) - (2)(0) = -3 - 0 = -3$
- $M_{13} = (1)(4) - (-1)(0) = 4 - 0 = 4$
- $|A| = 2(-5) - 3(-3) + 1(4) = -10 + 9 + 4$

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 0 & 4 & -3 \end{bmatrix}$$

the exact array being expanded alongside the working

Answer $|A| = 3$