

an equation built from derivatives, solved by separating, substituting, or multiplying by one integrating factor

Order and Degree

- a differential equation involves a function and its derivatives, solving it means finding the function
- order: the highest order of derivative that appears
- degree: the power of that highest derivative, written with no fractional or negative powers
- a derivative trapped inside a square root, log or sine has no defined degree until cleared

Order, Degree and Formation

order: the index of the highest derivative present

$order = n \text{ for } y^{(n)}$

degree: its power, once rationalised to polynomial form

$degree = p \text{ in } (y^{(n)})^p$

a family with n arbitrary constants needs n differentiations to remove them all

$n \text{ constants} \Rightarrow \text{order } n$

differentiate n times, then eliminate the constants between the relation and its derivatives, leaving

$\Phi(x, y, y', \dots, y^{(n)}) = 0$

the result must contain only x , y and derivatives of y , every constant gone

Order and degree, the special cases

a derivative sits under a radical, log or trig function $\rightarrow$ degree is undefined until the equation is cleared to polynomial form

a family carries n independent arbitrary constants $\rightarrow$ the differential equation formed has order n

every term shares the same total degree in x and y $\rightarrow$ the equation is homogeneous, substitute $y = vx$

P and Q are functions of y instead of x $\rightarrow$ use the sideways linear form $\frac{dx}{dy} + Px = Q$

Reading off order and degree *

Q. Find the order and degree of $x^2 \frac{d^2 y}{dx^2} + 3x \left(\frac{dy}{dx} \right)^2 + 4y = 0$.

- highest order derivative present: $d^2 y / dx^2$, second order
- it appears to the power 1, so degree is 1
- the lower-order $(dy/dx)^2$ term plays no part in either count

Answer **order 2, degree 1**

Forming a differential equation

Q. Form the differential equation of the family $y = Ae^{2x}$, A an arbitrary constant.

- differentiate once: $dy/dx = 2Ae^{2x}$
- the original gives $Ae^{2x} = y$, substitute directly
- $dy/dx = 2y$

Answer **$dy/dx - 2y = 0$**