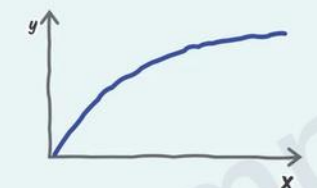


an equation about rates, solved by finding the whole family of curves it describes

What dy/dx actually is, on the curve itself



a solution curve that rises and levels off toward a limiting value, never overshooting it: the shape a cooling or bounded-growth equation's solution takes

a solution curve with one point marked: the curve's own slope there is the dy/dx the equation prescribes, point by point

solving a differential equation means finding the curve (or family of curves) whose tangent slope matches the rule the equation states

General vs particular solution *

General solution

carries exactly as many arbitrary constants as the order of the equation

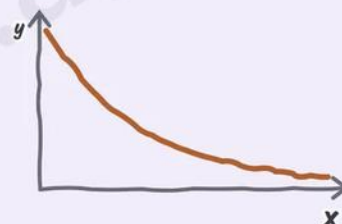
Particular solution

those constants pinned to numbers using given conditions, like a value of y at some x

an n th-order equation's general solution always has n arbitrary constants, never more, never fewer

Forming a differential equation from a family of curves

- start with the family, written with its arbitrary constants still in it
- differentiate as many times as there are constants
- eliminate the constants between the original equation and its derivatives, leaving one equation free of them



one member of the solution family to $dy/dx = -ky$, $k > 0$: it decays toward the axis, never touching it, and a different arbitrary constant draws a scaled copy of the same shape

the number of constants eliminated fixes the order of the resulting equation: eliminate one constant, get a first-order equation

Order and degree *

order: the highest derivative present

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0 \Rightarrow \text{order 2}$$

degree: that derivative's power, once the equation is a polynomial in every derivative

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = 0 \Rightarrow \text{degree 2}$$

degree is undefined here

$$\sqrt{\frac{d^2y}{dx^2}} + y = 0$$

the moment a derivative sits under a root or carries a fractional power, clear it first. If the equation still won't reduce to a polynomial in its derivatives, no degree exists at all

order comes from WHICH derivative appears; degree comes from ITS power, and only once every derivative sits free of roots and fractions