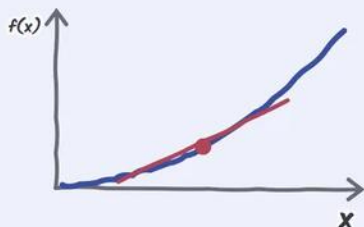


the rate of change at a single instant, built from a limit of a limiting ratio

Rate of change, and the definition of the derivative

*

- average speed is total distance over time taken
- the SPEED AT AN INSTANT is the limit of that ratio as the time shrinks to zero
- for f defined near a , if $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$ exists
- then f is differentiable at a , and this limit is $f'(a)$
- the process of finding a derivative is called differentiation; for a general x , $f'(x) = \frac{dy}{dx}$ where $y=f(x)$



a curve with a secant drawn across it and one point marked: the tangent is the limiting position this secant approaches as the second point slides in. whatever the shape, rising or falling, $f'(x)$ is the slope of that limiting line

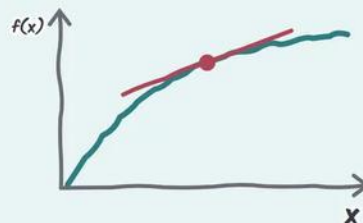
Left-hand and right-hand derivatives

- left-hand derivative at a : $Lf'(a) = \lim_{h \rightarrow 0^-} \frac{f(a+h)-f(a)}{h}$
- right-hand derivative at a : $Rf'(a) = \lim_{h \rightarrow 0^+} \frac{f(a+h)-f(a)}{h}$
- f is differentiable at a exactly when $Lf'(a) = Rf'(a)$; that common value is $f'(a)$

Worked: derivative of the square root, from first principle

Q. find the derivative of $f(x) = \sqrt{x}$ using the first-principle definition

- 1 $f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
- 2 multiply top and bottom by the conjugate $\sqrt{x+h} + \sqrt{x}$: the numerator becomes $(x+h) - x = h$
- 3 $\frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$, valid since $h \neq 0$ as the limit is approached
- 4 let $h \rightarrow 0$: $\frac{1}{\sqrt{x} + \sqrt{x}}$



a rising, flattening curve standing in for $y = \sqrt{x}$, cut by the line whose slope this limit measures

Answer $f'(x) = \frac{1}{2\sqrt{x}}$