



Gauss's law at work, then potential, dielectrics and the capacitor

Extending Gauss's law

- $\oint \mathbf{E} \cdot d\mathbf{s} = q_{\text{enc}} / \epsilon_0$, used for three symmetric shapes
- choose a Gaussian surface to match the symmetry: sphere, cylinder, pillbox
- flux from geometry = flux from Gauss's law: solve for E, no integration
- wire falls as $1/r$, shell as $1/r^2$ outside, the sheet does not fall

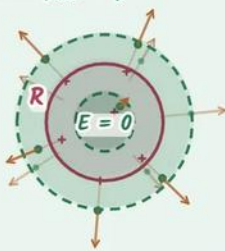
Field of a uniformly charged spherical shell

outside the shell, $r > R$
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

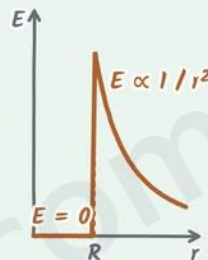
on the surface, $r = R$
$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 R^2}$$

inside the shell, $r < R$
$$E = 0$$

outside: $q_{\text{enc}} = q$



inside: $q_{\text{enc}} = 0$



a Gaussian sphere round the shell: $E = 0$ inside, like a point charge outside

from outside, the shell acts like a point charge q at its centre

Field of an infinite wire and an infinite sheet

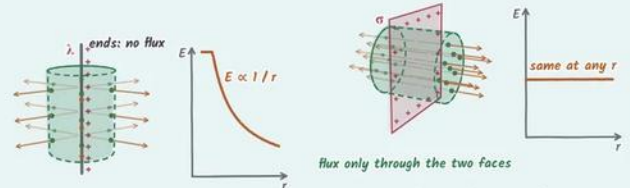


infinite line, density λ , cylindrical surface

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

infinite plane sheet, density σ , pillbox

$$E = \frac{\sigma}{2\epsilon_0}$$



a coaxial cylinder round the line: E falls as $1/r$

a pillbox through the sheet: E the same at any distance

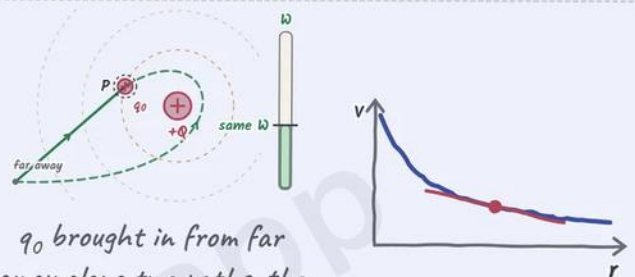
the sheet's lines run parallel and evenly spaced: E does not fall with distance

Electrostatic potential

potential energy per unit charge
$$V = \frac{U}{q}$$

potential difference, work done per unit charge
$$V_2 - V_1 = \frac{dW}{q}$$

field is the negative gradient of potential
$$E = -\frac{dV}{dx}$$



q_0 brought in from far away along two paths: the same work, so V at P is fixed

the slope of V against r , negated, is E at that r

the work does not depend on the path; $V = 0$ is chosen at infinity (or the earth)

