

compass and straight edge only, with the arcs left showing because they are the working

The rules of a construction

- only two instruments are allowed: a compass for arcs and circles, and an ungraduated straight edge for lines
- the ruler may be used to measure a given length when setting the compass, but never to mark a point by measuring along a line
- every arc must be left visible, because the arcs are the evidence that the figure was constructed and not merely drawn
- a protractor is not allowed, so an angle must be built from arcs, not measured out
- label every point as it is made, and write the steps beside the figure, since both carry marks
- a rough sketch of the finished figure drawn first is worth the thirty seconds: it tells you what to aim at

the base of most other constructions

Dividing a segment in a given ratio *

draw the segment AB of the given length
from A draw a ray AX making any acute angle with AB, on either side

along AX mark off $m + n$ equal arcs of any convenient radius, giving points A_1, A_2 and so on

for a ratio $m:n$, join the last point $A_{(m+n)}$ to B

through the point A_m draw a line parallel to that join, meeting AB at P

then P divides AB in the ratio $m:n$, which follows from the basic proportionality theorem

Why that construction works

- the arcs along the ray are equal by construction, so AX is divided into $m + n$ equal parts
- the line through A_m is parallel to the join from the last point to B
- by the basic proportionality theorem a line parallel to one side of a triangle divides the other two sides in the same ratio
- the ray is divided in the ratio $m:n$ at A_m , so AB must be divided in that same ratio at P
- the angle at A does not matter, and neither does the size of the arcs, which is why any convenient choice is allowed



a set of parallel lines cutting across two rays: equal spacings along one ray force proportional spacings along the other, which is the theorem that makes the division construction exact rather than approximate

given a scale factor m/n

Constructing a similar triangle

draw the given triangle ABC, and from A draw a ray AX at an acute angle to AB, on the side away from C

mark as many equal arcs along AX as the larger of m and n , giving A_1 up to that point

join A_n to B, where n is the denominator of the scale factor

through A_m draw a line parallel to A_nB , meeting the line AB at B'

through B' draw a line parallel to BC, meeting the line AC at C'

then triangle $AB'C'$ is similar to ABC with the ratio $m:n$, and it is larger when m exceeds n and smaller when it does not