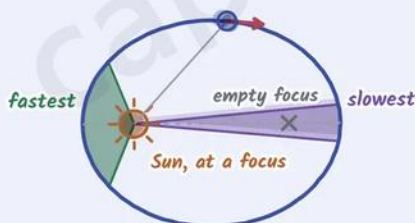


one inverse-square law, from Kepler's ellipses to why an astronaut floats

Kepler's three laws

- law of orbits: every planet moves in an ellipse with the Sun at one focus
- law of areas: line to the Sun sweeps equal areas in equal times
- this is just angular momentum conservation
- planet speeds up near the Sun, slows down far from it
- law of periods: $T^2 \propto r^3$, for every planet orbiting the same star



equal times sweep equal areas: short and wide near the Sun, long and thin far away

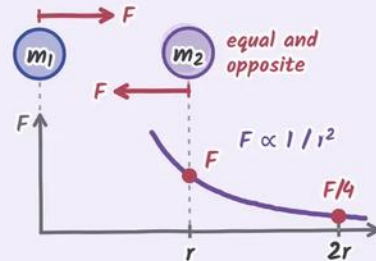
The universal law of gravitation

force between two point masses

$$F = \frac{Gm_1m_2}{r^2}$$

universal gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$



equal and opposite pulls along the line of centres; twice as far, a quarter of the pull

always attractive, along the line joining the masses; r is measured centre to centre. G is the same everywhere in the universe, whatever the masses or the medium

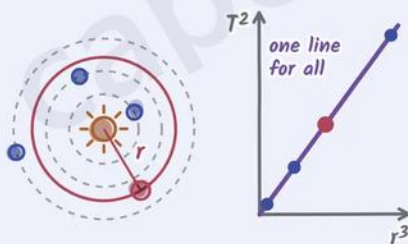
The law of periods

every planet of one Sun, same constant

$$T^2 \propto r^3$$

for a circular orbit of radius r

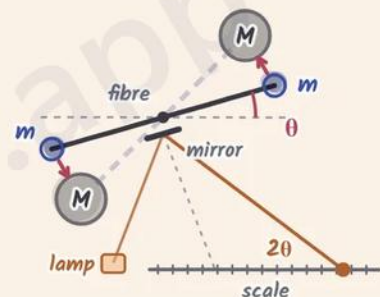
$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$



farther out means slower and a longer year; all planets lie on one straight line of T^2 over r^3
for an ellipse, r is the semi-major axis; the constant depends only on the Sun's mass

Measuring G , the Cavendish balance *

- a rod with small lead spheres m hangs from a fibre; big spheres M come near
- the rod twists until restoring torque matches gravitational torque
- $\tau = \frac{GmM}{r^2} L = K\theta$: measure the twist θ and get G
- a mirror on the fibre sends a light spot along a scale: it moves 2θ



seen from above: M pulls m , the rod twists through θ (drawn far larger than real)