

Indefinite Integration

differentiation run backward: a table of standard antiderivatives, then substitution, parts and partial fractions to reach it

Integration undoes Differentiation

- if $d/dx[g(x)] = f(x)$, then $\int f(x) dx = g(x) + c$
- c is the CONSTANT OF INTEGRATION: a whole family of antiderivatives differs only by a vertical shift
- sums integrate term by term, and a constant pulls straight out: $\int k f(x) dx = k \int f(x) dx$

Elementary Integration Formulae

power rule, n not -1 $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

power rule with a linear inner function

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{n+1} \cdot \frac{1}{a} + c$$

reciprocal

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \log|ax+b| + c$$

exponential

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

sine

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + c$$

cosine

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c$$

sec squared

$$\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + c$$

cosec squared

$$\int \operatorname{cosec}^2(ax+b) dx = -\frac{1}{a} \cot(ax+b) + c$$

every $(ax+b)$ entry is the plain formula with a $1/a$ tacked on, the chain rule showing up in reverse: the single most dropped factor under exam pressure

Special cases of the power rule *

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$n = -1$ → the power rule divides by zero, use log instead $\int x^{-1} dx = \log|x| + c$

$n = 0$ → the integrand is just 1 $\int 1 dx = x + c$

$a = 1, b = 0$ in the $(ax+b)^n$ → it collapses to the plain family power rule

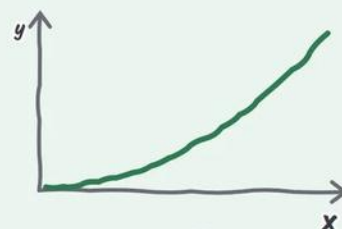
integrating a plain constant k → the result is a straight line in x $\int k dx = kx + c$

Power rule with a linear inner function

Q. Evaluate $\int (2x+3)^5 dx$.

① $n=5, a=2$: apply $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{n+1} \cdot \frac{1}{a}$

② $= \frac{(2x+3)^6}{6} \cdot \frac{1}{2}$



the general shape of the antiderivative $(2x+3)^6$: a curve rising away from its zero, curving upward

Answer $(2x+3)^6/12 + c$