

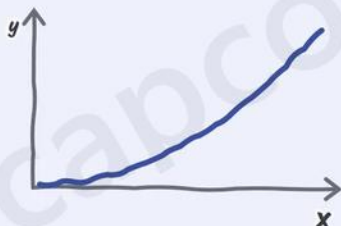
differentiation undone, one antiderivative at a time

Integration undoes differentiation

if F is an antiderivative of f $\int f(x) dx = F(x) + C$

standard power rule, for n not equal to -1 $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

the one exponent the power rule skips $\int \frac{1}{x} dx = \ln|x| + C$



the curve $y = x$ squared, used throughout this chapter's worked examples

every antiderivative of f differs from another only by a constant, which is why $+C$ is never optional

The constant of integration

- an indefinite integral is not one function, it's the whole family sharing the same derivative
- two antiderivatives of f differ only by a constant, since a constant's derivative is 0
- dropping $+C$ on a definite integral costs nothing; an indefinite one loses its whole family

Indefinite vs definite integral *

Indefinite, no limits

- a whole family of functions
- always carries $+C$

Definite, with limits

- a single number, $F(b) - F(a)$
- no $+C$, the limits fix it

More standard integrals

exponential $\int e^x dx = e^x + C$

the two inverse-trig integrals that show up constantly $\int \frac{1}{1+x^2} dx = \tan^{-1}x + C$

and the other one $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + C$



e^x : a curve that keeps steepening, never levelling off, and is its own derivative and its own integral

e^x is the only elementary function whose integral and whose derivative are both itself

Integration by substitution

pick a part of the integrand to call u $u = g(x)$

rewrite dx in terms of du $du = g'(x) dx$

the integral turns into one purely in u

$$\int f(g(x)) g'(x) dx = \int f(u) du$$

substitution works whenever the integrand pairs a function with something proportional to its own derivative

Integrate by substitution

Q. Evaluate the integral of $2x(x^2+1)^3$ with respect to x .

- Let $u = x^2+1$, so $du = 2x dx$
- the integrand $2x(x^2+1)^3 dx$ becomes $u^3 du$ exactly
- integrate: the integral of $u^3 du$ is $u^4/4 + C$
- substitute back $u = x^2+1$

Answer $(x^2+1)^4 / 4 + C$