

Introduction to Three Dimensional Geometry

one more axis, one more squared term, same two ideas

Three mutually perpendicular axes *

- space needs three coordinate axes, x , y , z , mutually perpendicular, meeting at origin O
- the three coordinate planes are XY (where $z=0$), YZ (where $x=0$) and ZX (where $y=0$)
- a point in space is the triple (x,y,z) : signed perpendicular distances from the three planes
- the origin is $(0,0,0)$; a point on the x -axis has $y=0$ and $z=0$ both

The eight octants

- three coordinate planes cut space into 8 octants, as two axes cut a plane into 4
- each octant is fixed by the SIGN of x,y,z : $(+,+,+)$ is the first, $(-,-,-)$ the opposite
- swapping the sign of even one coordinate moves a point into a different octant

Reading a point off its signs

$(+,+,+)$

first octant, all three coordinates positive

$(-,+,+)$

y and z stay positive, only x flips

$(+,-,-)$

only x stays positive

$(0,0,z)$

not an octant at all, this sits ON the z -axis itself

Distance between two points

$P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

distance from the origin

$$OP = \sqrt{x_1^2 + y_1^2 + z_1^2}$$



many paths can connect A and B , but the distance formula measures only the ONE straight path between them

this is the ordinary two-dimensional distance formula with one more squared term added, the same right-triangle idea one dimension further

Distance, built from a simpler idea



on a single line, distance between two points is just a subtraction. Each extra coordinate axis adds one more squared difference under the root

checking collinearity: three points A , B , C are collinear exactly when one of the three distances equals the sum of the other two