

six functions, six principal ranges, and the conditions on every identity



Why a restriction is needed at all *

- sine takes the value $1/2$ at infinitely many angles, so as it stands it is many to one and has no inverse
- the fix is to restrict the domain to an interval on which the function is one to one and still covers the whole range
- the interval chosen is called the principal branch, and the value it returns is the principal value
- so arc sin of $1/2$ is $\pi/6$ and NOT the whole family $n\pi + (-1)^n \pi/6$, which belongs to solving equations rather than to this function

The six, with domain and principal range *

Function	Domain	Principal range
arc sin x	$[-1, 1]$	$[-\pi/2, \pi/2]$
arc cos x	$[-1, 1]$	$[0, \pi]$
arc tan x	all reals	$(-\pi/2, \pi/2)$
arc cot x	all reals	$(0, \pi)$
arc sec x	$ x \geq 1$	$[0, \pi]$ without $\pi/2$
arc cosec x	$ x \geq 1$	$[-\pi/2, \pi/2]$ without 0

the ranges are the whole chapter: nearly every wrong answer here is a correct angle taken from outside the principal branch

Reading a principal value off the range

- for a negative argument, arc sin, arc tan and arc cosec return a NEGATIVE angle, since their ranges straddle zero
- arc cos, arc cot and arc sec return an angle in the SECOND quadrant instead, because their ranges are 0 to π
- so arc sin($-1/2$) is $-\pi/6$ while arc cos($-1/2$) is $2\pi/3$, and getting those two the same way round is the standard mistake
- the tidy way to state it: odd functions arc sin, arc tan and arc cosec satisfy $f(-x) = -f(x)$; the other three satisfy $f(-x) = \pi - f(x)$