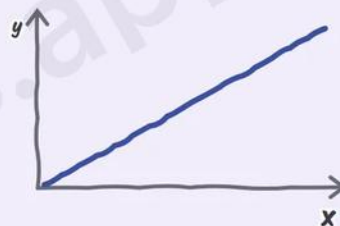


restrict the domain first, then the arrow can run backward

Why the domain must shrink first *



unrestricted, the curve rises then falls: one horizontal line crosses it twice, not one-one



restricted to a single monotonic stretch, it's one-one, and can now be inverted

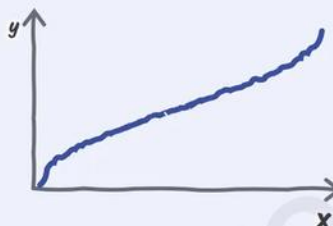
every trig function repeats forever; inverting needs exactly one branch, the principal value branch

$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$: principal branches

$$\sin^{-1}x \quad [-1,1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}x \quad [-1,1] \rightarrow [0,\pi]$$

$$\tan^{-1}x \quad \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

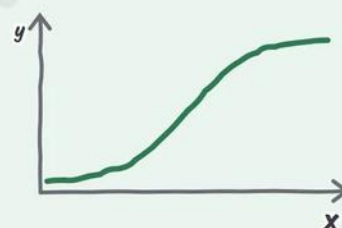


$\sin^{-1}x$'s principal branch: shallowest in the middle, steepening toward both ends

$\sin^{-1}$ and $\tan^{-1}$ share a range interval, but $\sin^{-1}$'s is closed $[\]$, $\tan^{-1}$'s open $(\)$: $\tan$ never reaches $\pm\pi/2$

Reading a principal branch

- each graph is just one strand chosen from the full periodic curve, cut so it's monotonic
- $\sin^{-1}x$ and $\tan^{-1}x$ are both increasing on their branch
- $\cos^{-1}x$ is decreasing, running from π down to 0 as x goes from -1 to 1



$\tan^{-1}x$'s branch: rising the whole way, flattening toward a limit at each end that it never reaches

$\sin^{-1}x$ vs $\cos^{-1}x$, same input

$\sin^{-1}x$

- odd function: $\sin^{-1}(-x) = -\sin^{-1}x$
- passes through the origin: $\sin^{-1}0 = 0$

$\cos^{-1}x$

- neither odd nor even: $\cos^{-1}(-x) = \pi - \cos^{-1}x$
- $\cos^{-1}0 = \frac{\pi}{2}$, not 0

linked: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for every x in $[-1,1]$; swap one for the other when it helps

restrict first, THEN invert. Every $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ you'll ever use already carries that restriction built in