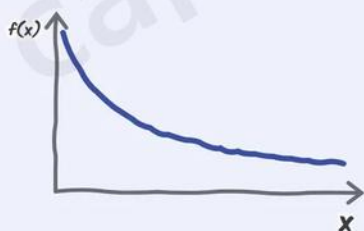


what a function does NEAR a point, and the slope that idea builds

What a limit means

- $\lim_{x \rightarrow a} f(x) = L$ means $f(x)$ gets arbitrarily close to L as x gets close to a , from either side. f itself need not even be defined AT a
- left-hand limit: $\lim_{x \rightarrow a^-} f(x)$, x approaches a from values LESS than a
- right-hand limit: $\lim_{x \rightarrow a^+} f(x)$, x approaches a from values GREATER than a
- the two-sided limit exists exactly when the left-hand and right-hand limits both exist AND are equal



as x grows, the curve approaches a fixed value without ever touching it: the same idea a limit captures at any point, not just at infinity

Algebra of limits

sum/difference

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

product

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \times \lim_{x \rightarrow a} g(x)$$

quotient, provided $\lim_{x \rightarrow a} g(x) \neq 0$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

constant multiple

$$\lim_{x \rightarrow a} [kf(x)] = k \lim_{x \rightarrow a} f(x)$$

every one of these rules needs BOTH individual limits to exist first. Apply them before checking existence and the answer can be nonsense

Worked: a limit by factoring *

Q. evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

- direct substitution gives $\frac{0}{0}$, undefined, so substitution alone won't work
- factor the numerator: $x^2 - 4 = (x - 2)(x + 2)$
- cancel the common factor, valid since $x \neq 2$ as the limit is approached, never reached: $\frac{(x - 2)(x + 2)}{x - 2} = x + 2$
- now substitute directly: $\lim_{x \rightarrow 2} (x + 2) = 2 + 2 = 4$

Answer the limit is 4

Limits of trigonometric functions *

the key one, x in radians

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

follows from the one above

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

a related limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

a different power of x

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

x MUST be measured in radians for every one of these. In degrees the same limit works out to a different, messier constant