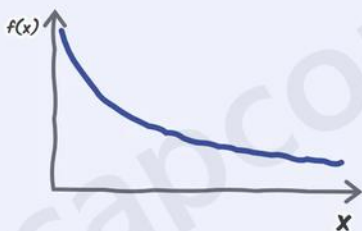




what $f(x)$ is heading toward as x closes in on a , from both sides at once

What a limit means *

- $x \rightarrow a$ means x takes values closer and closer to a , but never equal to a
- if a polynomial $P(x)$ is evaluated, $\lim_{x \rightarrow a} P(x) = P(a)$ directly: no special technique needed



a smooth curve: closing in on any x , $f(x)$ closes in on one height, and that height is the limit

One-sided limits *

left-hand, x to a from below

- notation: $\lim_{x \rightarrow a^-} f(x)$
- x approaches a through values LESS than a
- reads off behaviour just before a

right-hand, x to a from above

- notation: $\lim_{x \rightarrow a^+} f(x)$
- x approaches a through values GREATER than a
- reads off behaviour just after a

the two-sided limit exists, and equals that value, exactly when both sides agree

Algebra of limits

shorthand for this whole block

$$L = \lim_{x \rightarrow a} f(x), M = \lim_{x \rightarrow a} g(x)$$

sum, difference

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = L \pm M$$

product

$$\lim_{x \rightarrow a} [f(x)g(x)] = LM$$

quotient, provided $M \neq 0$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{L}{M}$$

constant multiple

$$\lim_{x \rightarrow a} [kf(x)] = kL$$

every rule here needs BOTH L and M to exist first; apply it before checking that and the result is meaningless

Quotient and infinity limits, the special cases

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}, \lim_{x \rightarrow \infty} \frac{P(x)}{Q(x)}$$

$$\lim_{x \rightarrow a} g(x) \neq 0$$

→ quotient rule applies directly

$$\lim_{x \rightarrow a} g(x) = 0, \lim_{x \rightarrow a} f(x) \neq 0$$

→ the limit does not exist

$$\lim_{x \rightarrow a} g(x) = 0 = \lim_{x \rightarrow a} f(x)$$

0/0 form: factor, rationalize, or use a standard limit

$$x \rightarrow \infty, \text{ equal degrees top and bottom}$$

→ limit is the ratio of leading coefficients

$$x \rightarrow \infty, \text{ numerator degree smaller}$$

→ limit is 0

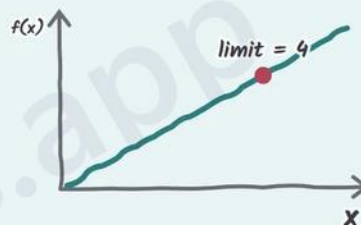
$$x \rightarrow \infty, \text{ numerator degree larger}$$

→ limit does not exist, grows unbounded

Worked: a 0/0 limit by factoring *

Q. evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

- direct substitution gives $\frac{0}{0}$, substitution alone will not settle it
- factor the numerator: $x^2 - 4 = (x - 2)(x + 2)$
- cancel the common factor, valid since $x \neq 2$:
$$\frac{(x - 2)(x + 2)}{x - 2} = x + 2$$
- now substitute directly: $2 + 2$



a rising line, the simplified $y = x + 2$: marked where x approaches 2 and f heads to 4

Answer the limit is 4