

a point and a direction build a line; a point and a normal build a plane

Vector Equation of a Line

- a line is fixed by a point and a direction: through $A(a)$ parallel to b , $AP = \lambda b$
- the PARAMETRIC form: $r = a + \lambda b$. λ is the parameter, each real value gives one point
- the same line, no parameter: $(r-a) \times b = 0$, the non-parametric form
- through TWO points $A(a)$, $B(b)$: $r = a + \lambda(b-a)$, since $b-a$ is a direction vector

Cartesian Forms of a Line

through (x_1, y_1, z_1) with direction ratios a, b, c

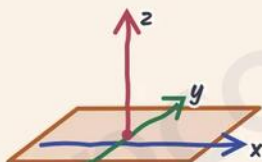
$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

through two points (x_1, y_1, z_1) and (x_2, y_2, z_2)

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

parametric form, from the symmetric form set equal to λ

$$x = x_1 + \lambda a, y = y_1 + \lambda b, z = z_1 + \lambda c$$



the three mutually perpendicular coordinate axes every line and plane in space is described against

the symmetric form needs a, b, c ALL nonzero; if even one is zero, use the parametric form instead

Vector and Cartesian equations of a line

Q. find the vector and Cartesian equations of the line through $(1, 2, 3)$ with direction ratios $2, -1, 2$

① vector form: $r = a + \lambda b$ with $a = \hat{i} + 2\hat{j} + 3\hat{k}$ and $b = 2\hat{i} - \hat{j} + 2\hat{k}$

② $r = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

③ Cartesian: $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

the point $(1, 2, 3)$, written as a 1 by 3 coordinate matrix

Answer

$$r = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k}), \quad \frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$

λ measures signed distance from the base point WHEN b is a unit vector: swap in direction cosines for direction ratios and $|\lambda|$ literally becomes the distance travelled along the line

Distance of a Point from a Line

- Line $r = a + \lambda b$, point $P(a)$ off it: drop perpendicular PM , M is AP 's projection onto b
- PM is found from right triangle AMP : $PM^2 = AP^2 - AM^2$, AM the scalar projection of $a-a$ along b

Distance-from-a-Line Formula *

distance of $P(a)$ from the line $r = a + \lambda b$

$$PM = \sqrt{|a-a|^2 - \left(\frac{(a-a) \cdot b}{|b|} \right)^2}$$

quicker route: $PM = \frac{|(a-a) \times b|}{|b|}$, the same distance from a cross product instead