

solving with $<$ and $>$ instead of $=$, one variable at a time and then two

What an inequality is

- a statement comparing two expressions with $<$, $>$, $\leq$ or $\geq$ instead of $=$
- a strict inequality ($<$, $>$) excludes the boundary value. a slack one ($\leq$, $\geq$) includes it
- solving one means finding every value of the variable that makes the statement true, not just one value
- the solution to a linear inequality in one variable is usually a whole ray or interval, never a single number

Rules for solving, same as an equation with one exception

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add or subtract the same number from both sides

$$a < b \Rightarrow a + c < b + c$$

multiply or divide both sides by a positive number

$$a < b, c > 0 \Rightarrow ac < bc$$

multiply or divide both sides by a negative number

$$a < b, c < 0 \Rightarrow ac > bc$$



$-3x < 12$ solved: $x > -4$, a hollow dot at -4 (excluded) and the ray running right for ever

the third rule is the whole chapter: the inequality sign flips whenever both sides are multiplied or divided by a negative number

Does the sign flip

what are you multiplying or dividing both sides by

a positive number

sign stays the same

$3x < 12$ divided by 3 stays $x < 4$

a negative number

sign flips

$-3x < 12$ divided by -3 becomes $x > -4$, not $x < -4$

adding or subtracting anything, positive or negative, never flips the sign. only multiplying or dividing by a negative one does

A compound inequality

Q. solve $-5 \leq 2x - 3 < 7$ for real x

- add 3 to every part: $-5 + 3 \leq 2x - 3 + 3 < 7 + 3$, giving $-2 \leq 2x < 10$
- divide every part by 2, a positive number so the signs stay put: $-1 \leq x < 5$



filled dot at -1 , included; hollow dot at 5 , excluded; every real number between is a solution

Answer x is any real number with $-1 \leq x < 5$

check an endpoint and a middle value: $x = -1$ gives $2(-1) - 3 = -5$, which satisfies $\leq$. $x = 5$ gives $2(5) - 3 = 7$, which fails the strict $<$, correctly excluded

Inequalities in two variables

- a linear inequality like $2x + 3y \leq 6$ is satisfied by a whole region of the xy -plane, not a curve
- the boundary line $2x + 3y = 6$ splits the plane into two half-planes. every point in one half satisfies the inequality, every point in the other doesn't
- a strict inequality ($<$ or $>$) draws the boundary as a dashed line, since points on it are excluded. $\leq$ or $\geq$ draws it solid
- test any one point not on the line, usually $(0,0)$, to see which half-plane is the solution



a boundary line, generic here: every point on the actual line $2x + 3y = 6$ would make the two sides exactly equal