

shade the feasible region, then check every corner: the optimum always sits at one

Linear Inequations and Half-Planes

- $ax+by+c=0$ splits the plane into the line, and two open half-planes, < 0 and > 0
- the half-plane with the origin is the origin side; the line is the boundary of both closed half-planes
- to solve $ax+by \leq c$: draw $ax+by=c$, test the origin off the line, shade its side if TRUE
- a line needs two points: $y=0$ for the x-intercept, $x=0$ for the y-intercept, then join

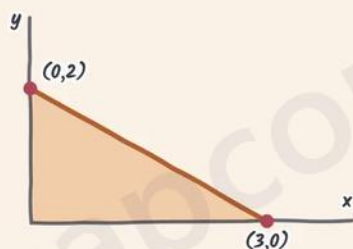
Convex Sets

- a set is convex if the segment joining ANY two of its points stays inside it
- a disc, a triangle's interior, a half-plane are convex; a crescent or star is not
- convex sets may be bounded (a polygon's interior) or unbounded (a half-plane)
- Theorem: the feasible region of a linear programming problem is always convex

Shading a region from a test point *

Q. represent the solution set of $2x+3y \leq 6$ graphically

- 1 draw $2x+3y=6$: $y=0$ gives $x=3$, so $(3,0)$; $x=0$ gives $y=2$, so $(0,2)$
- 2 the origin is off this line, so use it as the test point
- 3 substitute $(0,0)$: $2(0)+3(0)=0$, and $0 \leq 6$ is TRUE
- 4 the region is the side of the line containing the origin

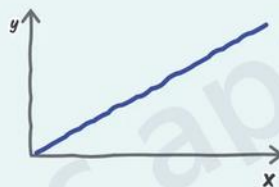


the boundary line $2x+3y=6$ through its own intercepts $(3,0)$ and $(0,2)$, a negative slope, with the origin side shaded since $(0,0)$ satisfies $2x+3y \leq 6$

Answer

shade the closed half-plane on the origin side of the line joining $(3,0)$ and $(0,2)$

Two lines, the pictures the rest of the chapter leans on



a linear constraint's boundary line: replace $\leq$ or $\geq$ with $=$, and the inequality only decides which SIDE of this line counts



two boundary lines closing in on each other: a corner of the feasible region is exactly where two constraint lines cross

Formulating a Linear Programming Problem

- decision variables: the unknown quantities being decided, usually x and y
- objective function: the linear expression in x, y being optimised, e.g. profit or cost
- constraints: linear inequations or equations the variables must satisfy, built from the resources stated
- non-negativity: $x \geq 0$ and $y \geq 0$, added whenever negative values make no physical sense