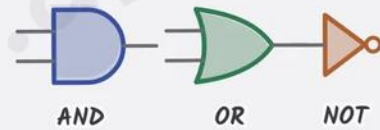


statements, connectives, and the algebra that simplifies both circuits and arguments

Statements and Logical Connectives

- a statement (proposition) is a sentence, definitely true or false, never both
- statements are denoted $p, q, r, \dots$
- questions, commands, requests, exclamations are never statements
- an open sentence like $x+6=9$ isn't one either, its truth changes with x
- a simple statement can't split into smaller ones
- a compound statement joins two or more simple ones with a connective
- 'and' gives conjunction $p \wedge q$
- 'or' gives disjunction $p \vee q$
- 'if...then' gives conditional $p \rightarrow q$
- 'if and only if' gives biconditional $p \leftrightarrow q$
- 'not' gives negation $\sim p$

Connectives as Logic Gate Symbols



the AND, OR, NOT gate symbols used in circuit diagrams, one per basic connective: AND for $\wedge$, OR for $\vee$, NOT for $\sim$

Negation of a statement

- negation $\sim p$ is true when p is false
- $\sim p$ is false when p is true
- negation of negation returns the original: $\sim(\sim p) = p$

Statement Patterns

- a proper combination of letters and connectives is a statement pattern
- also called a proposition, e.g. $p \rightarrow q, p \wedge q, \sim p \vee q$
- the letters used, here p and q , are its prime components
- the truth table lists the pattern's value for every combination of components
- n prime components need 2^n rows

Truth Tables *

Conjunction, $p \wedge q$

true only when both are true: $T \wedge T \equiv T$, else F

Disjunction, $p \vee q$

false only when both are false: $F \vee F \equiv F$, else T

Conditional, $p \rightarrow q$

false only when p is true, q is false: $T \rightarrow F \equiv F$, else T

Biconditional, $p \leftrightarrow q$

true when p and q match: $T \leftrightarrow T \equiv T, F \leftrightarrow F \equiv T$

these four hints replace the whole four-row table: check the one condition that decides it

Logical Equivalence *

- A and B are logically equivalent ($A \equiv B$) when truth tables match
- match means identical, row for row
- fastest check: build both truth tables under one set of p, q, r columns
- then compare the final columns directly