



row and column moves that build an inverse two ways, then solve equations with it



Elementary Transformations

- six elementary transformations: three on rows, three on columns, each giving a matrix **equivalent**, never equal
- interchange rows i and j : $R_i \leftrightarrow R_j$, also written R_{ij}
- the same move on columns: $C_i \leftrightarrow C_j$, written C_{ij}
- scale row i by $k \neq 0$: $R_i \rightarrow kR_i$; same on columns, $C_i \rightarrow kC_i$
- add k times row i to row j : $R_j \rightarrow R_j + kR_i$, only R_j changes
- the symbol $\sim$ means **equivalent to**, **never** equal: write $A \sim B$, not $A = B$

Notation for Elementary Transformations

interchange rows i and j

$$R_i \leftrightarrow R_j$$

scale row i by k , $k \neq 0$

$$R_i \rightarrow kR_i$$

add k times row i to row j

$$R_j \rightarrow R_j + kR_i$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

a 2 by 2 matrix labelled a_{ij} : i is the row a transformation acts on, j the column

columns use the identical three moves: replace every R with C in each rule above

Inverse by Transformation *

- if $AB=BA=I$ for square A , B is the inverse A^{-1} ; when it exists, it is unique
- for $AA^{-1}=I$: augment with I , reduce left to I ; same moves turn right into A^{-1}
- for $A^{-1}A=I$ instead, use column transformations throughout; **never mix** rows and columns in one reduction
- a row (or column) collapsing entirely to zero before I is reached means A is singular

A has rows $(2,1)$ and $(1,1)$

Inverse by Transformation, worked

augment A with I : left side has rows $(2,1)$ and $(1,1)$, right side has rows $(1,0)$ and $(0,1)$

$R_1 \leftrightarrow R_2$: left rows become $(1,1)$ and $(2,1)$, right rows become $(0,1)$ and $(1,0)$

$R_2 \rightarrow R_2 - 2R_1$: left rows become $(1,1)$ and $(0,-1)$, right rows become $(0,1)$ and $(1,-2)$

$R_2 \rightarrow -R_2$: left rows become $(1,1)$ and $(0,1)$, right rows become $(0,1)$ and $(-1,2)$

$R_1 \rightarrow R_1 - R_2$: left side is now I , right rows are $(1,-1)$ and $(-1,2)$

A^{-1} has rows $(1,-1)$ and $(-1,2)$

the SAME sequence of row moves that turns A into I turns I into A^{-1} : watch the right half, it is building the answer as you go

Minors and Cofactors

- minor M_{ij} of a_{ij} : the determinant left after deleting row i and column j
- cofactor $A_{ij} = (-1)^{i+j}M_{ij}$: same value as the minor, sign flipped when $i+j$ is odd
- signs alternate in a checkerboard, starting with plus at position $(1,1)$

