



Matrices



an algebra where order matters and division does not exist

What a matrix is, and the types worth naming

- a **matrix** is a rectangular array of numbers; a matrix of m rows and n columns has **order m by n** and mn entries
- **row matrix** has one row, **column matrix** one column, **square matrix** equal numbers of rows and columns
- **diagonal matrix**: every entry off the leading diagonal is zero; **scalar matrix**: diagonal with all diagonal entries equal; **identity I** : scalar with those entries all 1
- **zero matrix** has every entry zero, and it behaves like 0 for addition but NOT like 0 for multiplication, since a product of two non zero matrices can be zero
- **upper and lower triangular**: all entries below, or above, the leading diagonal are zero

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

a three by three matrix, entry a_{ij} in row i and column j , with the leading diagonal banded

Addition and scalar multiplication, which behave

- two matrices can be added only if they have the **same order**, and the addition is entry by entry
- addition is commutative and associative, the zero matrix is the identity, and every matrix has a negative
- **scalar multiplication** multiplies every entry, and it distributes over both matrix addition and scalar addition
- so under addition alone matrices behave exactly like numbers; everything surprising in this chapter comes from multiplication

Multiplication, and the three rules that follow from it

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- AB is defined only when the **columns of A equal the rows of B** , and the product then has the rows of A and the columns of B
- the (i, j) entry is the **row i of A dotted with column j of B** , so it is a sum of products and not an entry by entry operation
- AB is generally NOT BA , and one of the two may not even be defined
- $AB = 0$ does not force $A = 0$ or $B = 0$, so a matrix product has zero divisors and cancellation is illegal
- $AB = AC$ does not force $B = C$, unless A is invertible, and that exception is the whole reason inverses matter