



Matrices



a rectangular array with its own arithmetic: familiar rules, one big exception

A matrix, defined *

order (rows $\times$ columns)

$m \times n$

general entry, row i column j

$$A = [a_{ij}]$$

total entries in an $m \times n$ matrix

mn

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

a 2 by 3 matrix labelled a_{ij} : order states rows first, then columns

a matrix is just a rectangular array of numbers, arranged in rows and columns. Order always states rows first, then columns

Types, by shape

Row matrix

order $1 \times n$, a single row

Column matrix

order $m \times 1$, a single column

Square matrix

order $n \times n$, rows = columns

Zero matrix

every entry 0, any order, written 0

shape alone decides these four: no condition on the actual entries

Types, by what's inside a square matrix

Diagonal matrix

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

square, every off-diagonal entry is 0

Scalar matrix

diagonal, and every diagonal entry is the same number

Identity matrix I

scalar, with every diagonal entry = 1

identity $\subset$ scalar $\subset$ diagonal $\subset$ square: each one is a stricter version of the last

diagonal, scalar, identity are nested special cases, like matryoshka dolls: identity is the strictest, square is the loosest requirement here

Addition, subtraction, scalar multiplication

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addition, same order required

$$(A+B)_{ij} = a_{ij} + b_{ij}$$

subtraction

$$A-B = A+(-1)B$$

scalar multiple

$$(kA)_{ij} = k a_{ij}$$

addition and subtraction need matrices of the exact same order: a mismatched order simply can't be added, there's no rule that patches it

