

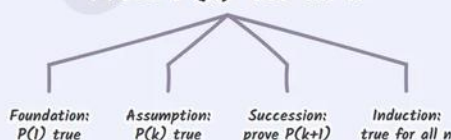
Methods of Induction and Binomial Theorem

prove it for one domino, prove it falls onto the next, and every domino after it is proved too

The four steps of induction

- Foundation: prove $P(n)$ is true for $n=1$ (checking $n=2, 3$ too is good practice, not required)
- Assumption: ASSUME $P(n)$ is true for $n=k$. Not proved, just the hypothesis the next step uses
- Succession: using that assumption, PROVE $P(n)$ is true for $n=k+1$
- Induction: conclude $P(n)$ is true for every positive integer n

Prove $P(n)$ for all n



the four steps of an induction proof, at a glance

Illustration: sum of the first n natural numbers

the statement $P(n)$ $1+2+3+\dots+n = \frac{n(n+1)}{2}$

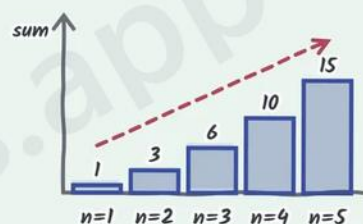


$P(k)$ and $P(k+1)$ as rungs; the succession step is the arrow UP from $P(k)$, the only thing the proof of $P(k+1)$ is allowed to lean on

Proving a summation formula by induction

Q. prove $1+2+3+\dots+n = \frac{n(n+1)}{2}$ for all $n \in \mathbb{N}$

- ① Foundation, $n=1$: LHS=1, RHS= $\frac{1(2)}{2} = 1$. Equal, so $P(1)$ is true
- ② Assumption: suppose $1+2+\dots+k = \frac{k(k+1)}{2}$
- ③ Succession: $1+2+\dots+k+(k+1) = \frac{k(k+1)}{2} + (k+1) = (k+1)(\frac{k}{2} + 1) = \frac{(k+1)(k+2)}{2}$
- ④ this is exactly $P(k+1)$. By induction, $P(n)$ holds for every $n \in \mathbb{N}$



the partial sums 1, 3, 6, 10, 15 for $n=1..5$, growing exactly as $n(n+1)/2$ predicts

Answer $1+2+\dots+n = \frac{n(n+1)}{2}$, proved

the succession step never re-derives the formula from scratch: it takes $P(k)$ as GIVEN and adds only the one new term $(k+1)$