



Motion in a Plane

two axes now, resolve first, then it's two straight lines



Scalar vs vector

Scalar

- only a size, no direction
- mass, time, speed, energy, temperature

Vector

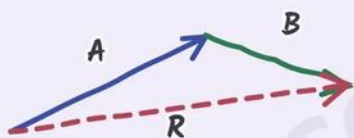
- size AND direction, both needed
- displacement, velocity, acceleration, force

a vector obeys the triangle law of addition, a scalar just adds like ordinary numbers

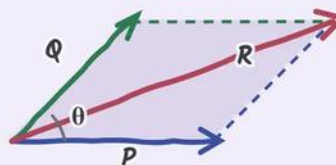
Special vectors

- equal vectors: same magnitude, same direction, starting point doesn't matter
- negative vector: same magnitude, opposite direction, $-A$ just reverses the arrow
- unit vector: magnitude 1, direction only, $\hat{A} = \frac{A}{|A|}$
- zero vector: magnitude 0, direction undefined, $A + (-A) = 0$

Adding vectors geometrically



triangle law: tail of B to head of A, R closes the triangle, $A+B=R$



parallelogram law: P and Q from one common tail, R is the diagonal through it

same answer either way, just a different picture. addition is commutative: $A+B = B+A$

Magnitude and direction of the resultant



resultant magnitude

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

direction, from A

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

subtraction

$$A - B = A + (-B)$$

R is biggest when $\theta=0$, smallest when $\theta=180$. it can never exceed $A+B$ or fall below $|A-B|$

Resultant: special angles *

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

same direction,
 $\theta=0^\circ$

→ vectors simply add

$$R = A+B$$

opposite, $\theta=180^\circ$

→ vectors subtract

$$R = |A-B|$$

perpendicular,
 $\theta=90^\circ$

→ cosine term vanishes

$$R = \sqrt{A^2 + B^2}$$

equal magnitudes, $A=B$

resultant bisects the angle between them

$$R = 2A\cos\frac{\theta}{2}$$

