

one axis, three quantities, four equations

Position

$$x(t)$$

where you are, with sign

$\rightarrow \frac{d}{dt}$

Velocity

$$v = \frac{dx}{dt}$$

how fast x changes

$\rightarrow \frac{d}{dt}$

Acceleration

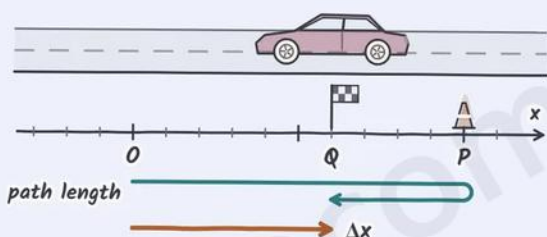
$$a = \frac{dv}{dt}$$

how fast v changes

integrate back: $v = \int a dt + C_1$, $x = \int v dt + C_2$, constants from initial values

Position, path length and displacement

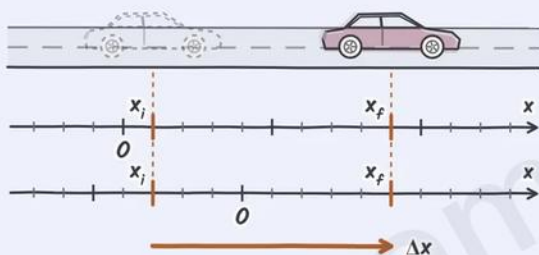
- position x : measured from a chosen origin O along the x -axis, its sign gives the side
- path length: the total distance covered along the actual path; a scalar, never negative
- displacement: $\Delta x = x_f - x_i$, the change in position, sign and all
- $|\Delta x| \leq \text{path length}$, equal only if the motion never turns back
- a round trip: displacement zero, path length not



there and back, O to P to Q : the path length counts every metre, Δx only start to finish

Moving the origin

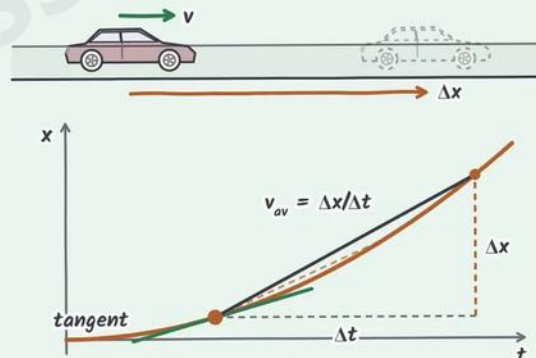
- positions x_i and x_f change when O moves
- displacement $x_f - x_i$ does not: it belongs to the motion



one drive, two origins: x_i and x_f change, $\Delta x = x_f - x_i$ does not

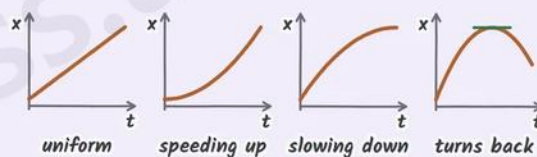
Velocity and speed

- average velocity: $\bar{v} = \frac{\Delta x}{\Delta t}$, the slope of the chord on $x-t$
- average speed = $\frac{\text{path length}}{\text{time}}$, never less than $|\bar{v}|$
- shrink Δt to zero: the chord turns into the tangent
- instantaneous velocity $v = \frac{dx}{dt}$, the slope of that tangent
- instantaneous speed is $|v|$: over a vanishing interval, path length = $|\Delta x|$



the chord's slope is v_{av} ; shrink Δt and it turns into the tangent, whose slope is v

Reading an $x-t$ graph



straight: uniform velocity. bending up: speeding up.

flattening: slowing down. a peak: it turns back

a horizontal $x-t$ line is rest. $x-t$ can never be vertical, that would be two positions at one time