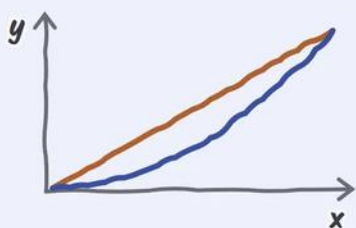


parabolas, exponentials, and what a shift does to an equation

## Nonlinear means the rate of change itself changes

- a linear function adds the same amount every step. A quadratic adds a growing amount, and an exponential multiplies by the same factor every step
- that one distinction answers most of the modelling questions: added each time is linear, multiplied each time is exponential
- a quadratic drawn on axes is a parabola, and every parabola has a vertex (its highest or lowest point) and an axis of symmetry through it
- the graph of a function and its equation carry the same information. The question decides which of the two is quicker to read



two functions from the same starting point. The straight one climbs by the same amount all the way across, while the curved one starts flatter and gets steeper: a fixed rate of change against a growing one

## One quadratic, three forms

standard form, and  $c$  is the  $y$ -intercept  $y = ax^2 + bx + c$

vertex form, vertex at  $(h, k)$   $y = a(x - h)^2 + k$

factored form,  $x$ -intercepts at  $p$  and  $q$   $y = a(x - p)(x - q)$

axis of symmetry, which is also the  $x$  of the vertex  $x = -\frac{b}{2a}$

sum of the two roots, with no factorising at all  $p + q = -\frac{b}{a}$

vertex form hands you the maximum or minimum, factored form hands you the roots, standard form hands you the  $y$ -intercept. Move toward whichever one the question is asking about

## Worked: intercepts and vertex

Q.  $h(x) = x^2 - 8x + 12$ . What is the sum of the  $x$ -intercepts of  $y = h(x)$ , and where is the vertex?

- 1  $x$ -intercepts are the  $x$  values with  $h(x) = 0$ :  $x^2 - 8x + 12 = 0$
- 2 factorise:  $(x - 2)(x - 6) = 0$ , so  $x = 2$  and  $x = 6$
- 3 their sum is 8, which is also  $-\frac{b}{a} = 8$ , so the factorising was optional for this part
- 4 the vertex sits on the axis of symmetry, midway between the roots:  $x = \frac{2 + 6}{2} = 4$
- 5 substitute back for the height:  $h(4) = 16 - 32 + 12 = -4$

Answer

the intercepts sum to 8, and the vertex is  $(4, -4)$

$a$  is positive here, so that vertex is the lowest point and  $-4$  is the minimum value of the function