

one restoring force proportional to displacement, and everything else in this chapter follows from it

Periodic, oscillatory and simple harmonic

- periodic motion repeats after a fixed interval, the period T
- oscillatory motion goes back and forth about a mean position
- every oscillation is periodic, not every periodic motion oscillates (circular motion)
- along a line: linear S.H.M.; along an arc: angular S.H.M.
- frequency $n = \frac{1}{T}$, angular frequency $\omega = 2\pi n = \frac{2\pi}{T}$

The defining condition and its equation

restoring force, always toward the mean position

$$F = -kx$$

Newton's second law gives

$$ma = -kx$$

the differential equation of linear S.H.M.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \omega^2 = \frac{k}{m}$$

that single condition, force proportional to displacement and pointed back home, is the whole definition of S.H.M., not just any back-and-forth swing qualifies

Displacement, velocity and acceleration

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displacement

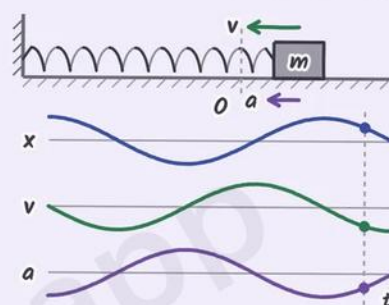
$$x = A \sin(\omega t + \phi)$$

velocity

$$v = \frac{dx}{dt} = A\omega \cos(\omega t + \phi) = \omega \sqrt{A^2 - x^2}$$

acceleration

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$$



v runs a quarter cycle ahead of x , and a is x turned upside down: a always points to the mean position

A is the amplitude, $\theta = \omega t + \phi$ is the phase, and ϕ (the phase at $t=0$) is fixed by wherever the observer starts the clock. The phase alone tells you the position, the direction of velocity and which swing the particle is in

The two extremes of a swing

	mean position, $x = 0$	extreme, $x = \pm A$
speed	largest, $A\omega$	zero, for an instant
acceleration	zero	largest, $A\omega^2$
energy	all kinetic	all potential