

Pair of Linear Equations in Two Variables

two lines on one grid, and the three things they can do

The standard form of a pair

first equation $a_1x + b_1y + c_1 = 0$

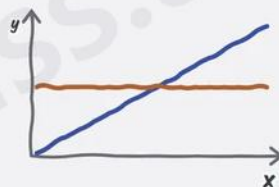
second equation $a_2x + b_2y + c_2 = 0$

neither may vanish completely $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$

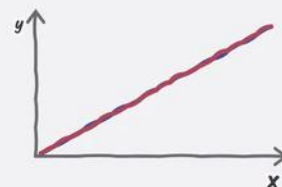
a solution is a point of the plane (x, y)

each linear equation in two variables draws a straight line. Solving the pair means finding whatever points the two lines have in common

Reading the lines off the grid



a slanting line and a level one: different slopes, so they cross at exactly one point, and that point is the single solution



both equations draw the same straight line, so the two pens run over each other and every point on the line solves the pair

the third case is the same slope at a different height: picture one of these lines lifted straight up without turning, so the gap never closes and the two never meet

What the ratios settle *

$$\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$$

a and b ratios differ $\rightarrow$ lines intersect: one solution, consistent $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

a and b ratios equal, c ratio differs $\rightarrow$ lines parallel: no solution, inconsistent $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

all three ratios equal $\rightarrow$ lines coincide: infinitely many, consistent and dependent $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

cross multiplication's own denominator is 0 $\rightarrow$ lines parallel or coincident, formula $a_1b_2 - a_2b_1 = 0$ gives nothing

the first three rows are checked against the three real pairs worked on the next page. Compare a then b then c: where the chain of equals first breaks is what names the case

Drawing the two lines

- two points fix a line: find two and rule it, no table of five needed
- the quickest two are the intercepts: set $x=0$ for one point, then $y=0$ for the other
- for $2x+3y=12$ that gives $(0, 4)$ and $(6, 0)$, and the line is drawn
- keep both coordinates whole numbers, or reading off the grid stops being exact

