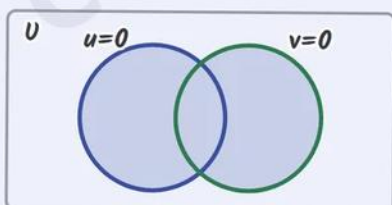


Pair of Straight Lines

one product equation hides two lines, and h squared minus ab says everything about how they sit

Combined Equation of a Pair of Lines

- let $u=a_1x+b_1y+c_1$, $v=a_2x+b_2y+c_2$; each of $u=0$, $v=0$ is an ordinary line
- $uv=0$ is the combined equation of $u=0$ and $v=0$, true exactly on either line
- $u=0$, $v=0$ are the separate equations; factorising the combined equation recovers them
- one product, two lines: $uv=0$ stands for **BOTH** lines, never their intersection point alone



the points satisfying $uv=0$ as a union: on $u=0$, or on $v=0$, or on both

lines $x+y-2=0$ and $2x-y+2=0$

Combined Equation, worked

the combined equation of $u=0$ and $v=0$ is $uv=0$, so multiply the two expressions directly

$$(x+y-2)(2x-y+2)=0$$

$$\text{expand: } x(2x-y+2)+y(2x-y+2)-2(2x-y+2)$$

$$=2x^2-xy+2x+2xy-y^2+2y-4x+2y-4$$

$$\text{collect like terms: } 2x^2+xy-y^2-2x+4y-4$$

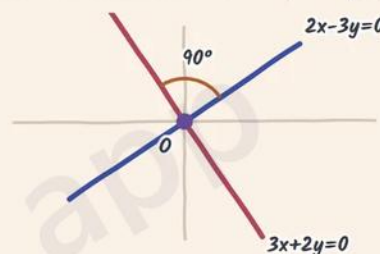
$$2x^2+xy-y^2-2x+4y-4=0 \text{ is the combined equation}$$

Separate equations from a combined equation

*

Q. find the separate equations of the lines represented by $6x^2-5xy-6y^2=0$

- split the middle term so each pair shares a common factor: $6x^2-9xy+4xy-6y^2=0$
- group: $3x(2x-3y)+2y(2x-3y)=0$
- factor out the common bracket: $(2x-3y)(3x+2y)=0$



the separate lines through the origin O , $2x-3y=0$ at slope $2/3$ and $3x+2y=0$ at slope $-3/2$: they meet at 90° , since $a+b=6-6=0$

Answer

the separate equations are $2x-3y=0$ and $3x+2y=0$

check by re-expanding: $(2x-3y)(3x+2y)=6x^2+4xy-9xy-6y^2=6x^2-5xy-6y^2$, matching the original

the symbol $uv=0$ is a **PRODUCT** being zero: exactly the rule 'if $AB=0$ then $A=0$ or $B=0$ ' from ordinary algebra, applied to two linear expressions instead of two numbers