

Permutations and Combination

order matters for a permutation, order is thrown away for a combination

Two fundamental principles of counting

- addition principle: SEPARATE operations in m or n ways, no overlap: EITHER one is $m+n$ ways
- multiplication principle: one operation in m ways, then another in n ways: BOTH is $m \times n$ ways
- 'OR' signals addition; 'AND' or 'followed by' signals multiplication
- invariance principle: the count never depends on the order the choices are made or listed

Factorial notation

definition, n a positive integer

$$n! = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$$

by convention

$$0! = 1$$

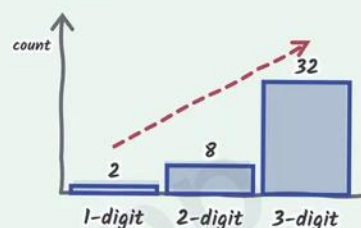


a tree splits a set into disjoint branches: choosing ONE game from either is $3+4=7$ ways, the addition principle

Addition then multiplication together *

Q. how many even numbers, formed from the digits 2, 3, 7, 8, are less than 1000?

- 1-digit even numbers: only 2 or 8 qualify, 2 ways
- 2-digit even numbers: unit's place even (2 ways), ten's place any of 4 digits: $4 \times 2 = 8$ ways
- 3-digit even numbers: unit's place even (2), ten's and hundred's place any (4 each): $4 \times 4 \times 2 = 32$ ways



the three case counts, 2, 8, 32, added together for the answer 42

Answer $2+8+32=42$

the three cases (1-digit, 2-digit, 3-digit) are mutually exclusive, so their counts are ADDED; within each case the digit-by-digit choices are MULTIPLIED

