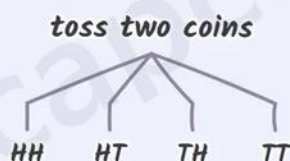


counting without listing, whether order matters or not

The fundamental principle of counting

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- if one task can be done in m ways and, after it's done, a second task can be done in n ways, the two together can be done in $m \times n$ ways
- this multiplication extends to any number of tasks done in sequence: multiply the count at every stage
- if either task can be done, not both, the counts add instead: $m+n$ ways



2 outcomes for the first coin, 2 for the second, $2 \times 2 = 4$ outcomes total, one branch per outcome

Factorial notation

n factorial: the product down to 1

$$n! = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1$$

by definition

$$0! = 1$$

one factorial built from the next smaller one

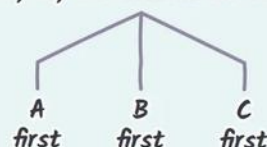
$$n! = n \times (n-1)!$$

$0! = 1$ is a definition, not something to derive. it's what keeps the nPr and nCr formulas working cleanly at the edges

Permutations: arrangements where order matters

- a permutation of r objects out of n distinct ones is an ordered selection: swapping two chosen objects gives a different permutation
- read nPr as 'the number of ways to arrange r objects chosen from n distinct objects, order counted'
- when $r=n$, every object is placed: the count is just $n!$

eg: A, B, C: choice of first letter



3 choices for the first letter. each one still leaves 2 letters to place after it, not drawn here

The permutation formula

r objects chosen from n , order matters ${}^n P_r = \frac{n!}{(n-r)!}$

all n objects arranged ${}^n P_n = n!$

this is just the fundamental counting principle written compactly: n choices for the first spot, $n-1$ for the next, down to $n-r+1$ for the last

Combinations: selections where order doesn't matter

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- a combination of r objects out of n is an unordered selection: a committee of A, B and C is the same committee as C, A and B
- every combination of r objects can be arranged in $r!$ different orders, and each of those orders is a separate permutation
- so permutations overcount combinations by exactly a factor of $r!$