

expressions built from whole number powers, how they divide, and the theorems that save the division

What counts as a polynomial

- a polynomial in x is a sum of terms of the form ax^n where every n is a whole number and a is a real coefficient
- so $3x^2 - 5x + 2$ is a polynomial, but x^{-1} and $\sqrt{x}$ are not, because their powers are not whole numbers
- a variable in a denominator disqualifies the expression too, since that is a negative power in disguise
- the degree is the highest power of the variable appearing with a non zero coefficient
- the leading coefficient is the number multiplying that highest power, and the constant term is the one with no variable
- a zero polynomial is just 0, and its degree is not defined

Naming a polynomial two ways

Degree	Called	Example
0	constant	7
1	linear	$2x + 3$
2	quadratic	$x^2 - 5x + 6$
3	cubic	$x^3 + 2x - 1$
4	biquadratic	$x^4 - 16$

polynomials are also named by how many terms they have: one term is a monomial, two a binomial and three a trinomial. Degree and term count are separate descriptions

The identities that do most of the work

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square of a sum

$$(a+b)^2 = a^2 + 2ab + b^2$$

square of a difference $(a-b)^2 = a^2 - 2ab + b^2$

difference of two squares $a^2 - b^2 = (a+b)(a-b)$

cube of a sum

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

sum of two cubes

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

difference of two cubes

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$\begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$$

a square of side a plus b cut into four pieces: one of area a^2 , one of area b^2 , and two identical rectangles of area ab . Those two rectangles are the $2ab$ term, which is why the square of a sum is never just a^2 plus b^2

in the cube factorisations the sign inside the bracket is opposite to the sign outside for the middle term, which is the detail most often written the wrong way round